Seismicity models based on static stress triggering

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Landers sequence
Hainzl et al. JGR 2009
... what everybody knows:

Colorful static stress maps showing generally some correlation between

- aftershock locations and positive stress lobes
- reduced activity and stress shadows

King et al. BSSA 1994

Stein Nature 1999
**Goal** of this lecture is to understand and discuss quantitative predictions of the static stress triggering model:

1. Underlying assumptions
2. Predictions of the model
3. Potentials & Limitations (due to unknowns/uncertainties) of model applications

**Outline:**

1. Neglect timing of earthquakes - A simple clock-advance model:
   - spatial aftershock distribution & decay
   - aftershock productivity
2. Time-evolution assuming rate-state-dependent friction:
   - simple considerations
   - examples of model applications
Coulomb Failure Stress (CFS) change:

\[ \Delta \text{CFS} = \Delta \tau - \mu \Delta \sigma_n \]

- \( \tau \): shear stress
- \( \sigma_n \): normal stress (including pore pressure)
- \( \mu \): friction coefficient

Modified from Hill, BSSA 2008
Origin of stress changes can be:

(1) **coseismic**:
- dynamic (seismic waves)
- static (permanent)

(2) **aseismic**:
- pore pressure changes
- continuous or transient creep
- visco-elastic deformation
- dike intrusion
- …
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M7.2 2008 Iwate-Miyagi Nairiku (IWTH24)
Let's start with simplest assumptions:

1. **Constant stress build-up due to tectonic loading**

2. **Existence of a population of faults where earthquake can occur**

3. **Without any stress disturbance:**
   - Earthquake rate is constant
   - Poisson model (assumption of seismic hazard assessment)
Critical stress at which earthquake nucleation starts

Clock advance - time needed to achieve ΔCFS by tectonic loading: \[ \Delta t = \Delta CFS / \dot{\tau} \]

Number of triggered events: \[ N_a = r \Delta t = (r / \dot{\tau}) \Delta CFS \] if ΔCFS>0

else \[ N_a = 0 \]
Number of triggered events $\sim \Delta CFS$ \hspace{1em} (if $\Delta CFS>0$)

Proportionality factor: 
\[
c = \frac{r}{\dot{\tau}} \approx \frac{V}{M_0}
\]

**Ratio between seismogenic volume and the average seismic moment per earthquake**

**Kostrov 1974:**
\[
\dot{\tau} = 2G \epsilon_{ij} = \lim_{\Delta t \to \infty} \frac{1}{V \Delta t} \sum_{k=1}^{N} M_{0ij}^k
\]
\[
= \lim_{\Delta t \to \infty} \frac{N}{V \Delta t} \overline{M_{0ij}} = \frac{r}{V} \overline{M_{0ij}}
\]
\[
\approx \frac{r}{V} \overline{M_0}
\]

Assumption: Similar earthquake mechanisms
For Gutenberg-Richter distributed earthquakes within \([M_{\text{min}}, M_{\text{max}}]\):

\[
\text{pdf}(M) = \ln(10) b \frac{10^{-b(M-M_{\text{min}})}}{1-10^{-b(M_{\text{max}}-M_{\text{min}})}}
\]

Average seismic moment per earthquake (NO time average!):

\[
\overline{M}_0 = \int_{M_{\text{min}}}^{M_{\text{max}}} \text{pdf}(M) 10^{9.1+1.5M} \, dM
\]

\[
= 10^{9.1+1.5M_{\text{min}}} \frac{b}{1.5-b} \frac{10^{(1.5-b)(M_{\text{max}}-M_{\text{min}})}-1}{1-10^{-b(M_{\text{max}}-M_{\text{min}})}}
\]

Proportionality factor \(c=V/\overline{M}_0\) can be calculated for given \(b\) and \(M_{\text{max}} (M_{\text{min}})\).
\[ N_a (x) \sim \Delta CFS(x) \quad \text{(for } \Delta CFS > 0) \]
Model Prediction:
No earthquakes in regions with $\Delta CFS<0$ (stress shadows)

Observation:
A significant fraction of earthquakes occur in stress shadows ... is static stress triggering not working?

Not necessarily because of:
1. uncertainty of slip inversions
2. unresolvable small scale slip
3. variable receiver mechanisms
4. secondary stress changes
Small scale slip variability

→ can explain on-fault activation
Variable receiver mechanisms

Faults with different orientations always exist.

Examples:

"unique" mechanism: $\Delta \text{CFS} < 0$  
→ no aftershocks

mean mechanism: $\Delta \text{CFS} < 0$ but some faults have $\Delta \text{CFS} > 0$  
→ aftershocks ~ blue area
Variable receiver mechanisms

fixed mechanism

variable mechanisms (+-10°)

z=7km
Variable receiver mechanisms

fixed mechanism

variable mechanisms (+-30°)

NO absolute stress shadows, only regions with reduced activation!
Distance decay

**Theory** (homogeneous elastic full-space):

Far-field:  
- dynamic stress $\sim \frac{1}{r^2}$
- static stress $\sim \frac{1}{r^3}$

*Felzer & Brodsky, Nature 2006:*

$M_{\text{main}} = 2-3$

$M_{\text{main}} = 3-4$

$\rho(r) = cr^{-1.35 \pm 0.12}$

$\rho(r) = cr^{-1.37 \pm 0.1}$

Static stress cannot be the driving force
Distance decay

Theory for homogeneous elastic full-space:

Far-field:  
- Dynamic stress $\sim 1/r^2$
- Static stress $\sim 1/r^3$

Richards-Dinger et al., Nature 2010:

Improper selection of "aftershocks" does not allow this conclusion.
Distance decay

Theory for homogeneous elastic full-space:

Far-field: dynamic stress $\sim 1/r^2$
static stress $\sim 1/r^3$

Marsan & Lengliné, JGR 2010:

... with an improved statistical attempt to isolate earthquakes causally related to the mainshock:

Exponent = 1.7-2.1
Separation based on the smallest space-time distance

\[ n_{ij} \sim \Delta t^d 10^{-bM} \]

between an event and all preceding events, where \( d \) is the assumed fractal dimension.

Method based on

Biasi & Paczuski 2004
Zaliapin et al. 2008
\[ P_m(r) = \begin{cases} 
\alpha \frac{qr^\gamma}{\left(\frac{r^{\gamma+1}}{L_m^{\gamma+1}} + 1\right)^{1+\frac{q}{\gamma+1}}} & \text{if } r < 10\text{km}, \\
\beta \frac{dr^\gamma}{\left(\frac{r^{\gamma+1}}{L_m^{\gamma+1}} + 1\right)^{1+\frac{d}{\gamma+1}}} & \text{if } r > 10\text{km}.
\end{cases} \]

\( y = 0.6 \)
\( q = 0.35 \)
\( d = 1.2 \)

\(~ r^{0.6} ~)
\(~ r^{-1.35} ~)
\(~ r^{-2.2} ~)

\( M_{\text{main}} = 3-4 \)

- all
- \( \Delta t < 1 \) hour

3 scaling regimes

Southern California
... from empirical observations back to static stress-triggering:

**Analysis for synthetic mainshock ruptures:**

- select magnitude M
- uniform (or fractal) slip
- empirical relations between M and slip/area
- epicenter randomly chosen within rupture area
- effective friction coefficient: $\mu = 0.5$
- elastic half-space (or layered half-space)
Aftershocks in the hypocenter depth layer:

Far field: \( \Delta CFS \sim r^{-3} \)
\( A \sim r \)

\( \rightarrow N_a \sim r^{-2} \)

A \( \sim (r+\Delta r/2)^d - (r-\Delta r/2)^d \)

\( A \): seismogenic area at distance \( r \)
\( d \): fractal fault dimension (uniform: \( d=2 \))

Linear aftershock density:

\( P(r) \sim \overline{\Delta CFS}(r) H(\Delta CFS) \frac{A}{\Delta r} \)
... but aftershocks occur not only at the hypocenter depth layer:

**Integration over depth interval**

![Graph showing probability density (pdf) vs. epicentral distance (km)]

- **995-1005 km**: grey shaded area
- **3-13 km**: red line

- **M=2.5**

- **much slower decay!**

- **r^{-2}**
Adaption to Southern California seismicity by random selection of depth & focal mechanism

Yang, Hauksson & Shearer (2012)

adapted model forecast
\( d=2, \) uniform
Adaption to Southern California seismicity by random selection of depth & focal mechanism

Yang, Hauksson & Shearer (2012)

Adapted model forecast
\(d = 1.8\), fractal

empirical

\(M = 2.5\)
Independent evaluation by means of the ETAS model:

Is $d=1.8$ a reasonable value for background activity?

Method based on Beauval et al., BSSA 2006
Scaling of the correlation integral: scaling exponent = fractal dimension

$R^{1.6}$

$R^{1.8}$

$N(r < R) / N(r < \infty)$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$10^{-1}$

$10^{0}$

$10^{1}$

$10^{2}$

$d=1.8$ in agreement with California seismicity
Productivity of aftershocks

\[ N_a = \int_0^\infty \Delta \text{CFS}(r) H(\Delta \text{CFS}) \ A(r) \ \frac{W}{M_0} \ d\tau \]

- \( W = 10 \text{ km} \)
- \( b = 1.0 \)
- \( M_{\text{min}} = -2.0 \)
- \( M_{\text{max}} = 7.5 \)

California: *Helmstetter et al., JGR 2005*
Largest aftershock magnitude as another way to compare with observations:

Empirical observation: $\langle M_{a,\text{max}} \rangle = M_m - 1.2$ (Bath, 1965)
... thus static stress triggering can explain first-order aftershock characteristics regarding total numbers and spatial distribution.

→ static stress changes seem to be the major driving force for aftershock nucleation!

However, the model misses so far the timing of aftershocks and the contribution of aftershock induced secondary stress changes
One possibility: A statistical model extension

**modified ETAS-model:**

... where productivity scaling and spatial kernel are replaced

\[
R(t, \mathbf{x}) = \mu + \sum_{i: t_i < t} K_0 e^{\alpha M_i} \cdot \frac{1}{(c + t - t_i)^p} \cdot \frac{c_{\text{norm}}}{(r^2 + d^2)^{\gamma/2}}
\]

*Ogata, 1988*

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**without slipmodel**

**with slipmodel**

*Bach & Hainzl, JGR 2012; Zakharova et al., JGR 2013*

... currently tested in CSEP New Zealand
Another possibility:

Rate- and state-dependent frictional nucleation model

Dieterich, JGR 1994

from lab-derived friction law
to statistical response of fault populations

\[
\begin{align*}
R &= r / \dot{\tau} \gamma \\
& \text{stress step: } \Delta S \\
R(t, \Delta S) &= \frac{r}{\Delta S} \frac{t}{A \sigma_n} & 1 + \left( e^{\Delta S / A \sigma_n} - 1 \right) e^{-t / t_a}
\end{align*}
\]

- \(r\): background rate
- \(A\): friction parameter
- \(\dot{\tau}\): tectonic stressing rate

\(t_a = \frac{A \sigma_n}{\dot{\tau}}\) relaxation time
Response to a stress step of $\pm \Delta CFS$:

\[
\text{# triggered} (+\Delta CFS) = \text{# missed} (-\Delta CFS)
\]

\[
\Delta CFS = +10 \, \Delta \sigma
\]

\[
\Delta CFS = -10 \, \Delta \sigma
\]

\[
\Delta CFS = \text{all}
\]
In the case of different stressing rates:

\[ N_{\text{tot}} = \left( \frac{r}{\dot{\tau}} \right) \Delta \text{CFS} \quad \text{(same as in the clock advance model!)} \]

\[ \# \text{ triggered} = \# \text{ triggered} \quad \text{if} \quad \frac{r}{\dot{\tau}} = \text{constant} \]
**Advantage:** Earthquake rate can be determined for complex stress histories resulting from aseismic & coseismic processes.

**Example:**

- **rainfall**
- **pore pressure**
- **diffusion**

Earthquake activity at Mt. Hochstaufen, SE Germany

*Hainzl et al. GRL 2006*

*Hainzl et al. JGR 2013*
**Advantage:** Earthquake rate can be determined for complex stress histories resulting from aseismic & coseismic processes.

**Example:** rainfall

Earthquake activity at Mt. Hochstaufen SE Germany

Hainzl et al. GRL 2006
Hainzl et al. JGR 2013
Challenge: Large uncertainties of $\Delta$CFS-calculations

- without uncertainties
- variable receiver faults
- finite cell volume
- small scale slip
- alternative slip models

M9 Tohoku aftershock sequence

Model integration by Monte-Carlo sampling ... Cattania et al. JGR 2014
Model performance improves significantly if uncertainties are systematically taken into account:

\[ \text{Without uncertainties} \]

\[ \text{With uncertainties} \]

M9 Tohoku aftershock sequence

Cattania et al. JGR 2014
Summary

Static stress triggering can explain first-order characteristics:
- spatial distribution
- productivity
- Omori aftershock decay

However:

Uncertainties of the actual stress state (use of Poisson model for the initial state) and the ΔCFS-calculations are large and limit the forecast ability.

Proper accounting for uncertainties is necessary for hypothesis testing and improved forecasts.
Thank you!

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https://statsei9.quake.gfz-potsdam.de