Earthquake nucleation

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How do earthquakes start?

Do small and large earthquakes start differently?
Predictive value of earthquake onset and foreshock sequences?

• Seismological observations: seismic nucleation phase, foreshock sequences
• Laboratory observations
• Friction laws and earthquake nucleation
• Further implications: stress drop, recurrence time, seismicity rate, tremors
Seismological observations

Ellsworth and Beroza (1995)
Beroza and Ellsworth (1996)
Seismological observations

Foreshock sequences

Dodge et al (1996)

Mainshock Moment vs Foreshock Radius

- Area enclosing ruptures
- Area enclosing hypocenters
- Radius of nucleation region from Ellsworth & Beroza, 1995

(a) Slow Cascade

(b) Pre-slip Triggering
A Mw3.9 earthquake in Alaska triggered by Love waves from the April 11, 2012 Mw 8.6 Sumatra earthquake

Tape et al (2013)
Seismological observations

Nucleation phase of the Mw3.9 Alaska triggered earthquake
Tape et al (2013)
Seismological observations

\[ \tau_c^2 = 4\pi^2 \frac{\int_0^{\tau_0} u^2(t)dt}{\int_0^{\tau_0} u^2(t)dt} \]

\(1/\tau_c \sim \) instantaneous frequency


Nakamura (1988)
Seismological observations

Magnitude dependence of early dominant period

\[ \tau_c^2 = 4\pi^2 \frac{\int_0^{\tau_0} u^2(t) \, dt}{\int_0^{\tau_0} u^2(t) \, dt} \]

Seismological observations

Peak ground displacement (Pd) grows exponentially.
Growth rate depends on magnitude

Colombelli et al (2014)
Seismological observations

Foreshock sequence of the 2011 Tohoku earthquake

Seismological observations

Foreshocks of the 2014 Iquique (Chile) earthquake
Laboratory experiments

Ohnaka (1990)
Laboratory experiments

Laboratory experiments

Laboratory foreshocks

Rubinstein et al (2007)
Laboratory experiments

Foreshocks promoted by aseismic slip

McLaskey and Kilgore (2014)
Fracture mechanics

Static equilibrium in a linear elastic solid with a crack and boundary conditions:
\[ \sigma(x) = \sigma_0 \text{ for } |x| > a \quad \text{and} \quad \sigma(x) = 0 \text{ for } |x| < a. \]

Stress singularity at the crack tips
Fracture mechanics

Stress singularity at the crack tips.
Asymptotic form:

\[ \sigma = \frac{K_1}{\sqrt{2\pi r}} + O(\sqrt{r}) \]

where \( r \) is the distance to a crack tip,
\( K \) is the stress intensity factor
and \( \Delta \sigma \) the stress drop (here, \( \sigma_0 - 0 \))

In reality, stresses are finite: singularity accommodated by inelastic deformation.
Fracture mechanics

Energy release rate = energy flux to the crack tip per unit of crack advance:

$$G(a) = \frac{K(a)^2}{2\mu} = \pi \frac{\Delta \tau^2 a}{2\mu}$$

During quasi-static crack growth, this energy is dissipated at the crack tip into fracture energy

$$G(a) = G_c$$

During dynamic growth, $G(a) > G_c$
At rest, $G(a) \leq G_c$
Nucleation size

At the onset of rupture (critical equilibrium): \( G_c = \pi \frac{\Delta \tau^2 a}{2\mu} \)

→ earthquake initiation requires a minimum crack size (\textit{nucleation size})

\[
a_c = \frac{2\mu G_c}{\pi \Delta \tau^2}
\]

(\( \mu \sim 30 \text{ GPa}, \Delta \tau \sim 5 \text{ MPa} \))

Estimates for large earthquakes: \( G_c \sim 10^6 \text{ J/m}^2 \) → \( a_c \sim 1 \text{ km} \)

... how can M<4 earthquakes nucleate ?!

Laboratory estimates: \( G_c \sim 10^3 \text{ J/m}^2 \) → \( a_c \sim 1 \text{ m (M-2)} \)

→ \( G_c \) scaling problem
Limitations of fracture mechanics:
Rock strength is finite

Shear stress (Mpa)

Normal stress (Mpa)

Byerlee’s law
\[ \tau \sim 0.6\sigma \]
Fault zone damage

Fault zone thickness and maturity

(Savage and Brodsky, 2011)
Cohesive zone models

Assumption: dissipative processes are mapped onto the fault plane, represented by a distribution of cohesive stresses near the crack tip

Usual cohesive models:
- constant (Dugdale, Barenblatt)
- linearly dependent on distance to crack tip (Palmer and Rice, Ida)
- linearly dependent on slip (Ida, Andrews)
Cohesive zone size

- Cohesive stresses generate a negative stress intensity factor

\[ K_c \sim -(\tau_s - \tau_d)\sqrt{\Lambda} \]

that cancels the singularity: \( K + K_c = 0 \)

- Size of the cohesive zone \( \Lambda \sim \frac{K^2}{(\tau_s - \tau_d)^2} \)

- \( G_c = \frac{K^2}{2\mu} \)

\[ \Lambda \sim 2\mu G_c/(\tau_s - \tau_d)^2 \]
Laboratory-derived friction laws

Requirements:
• High normal stress (100 MPa)
• High slip rate (1 m/s)
• Large displacements (>1 m)
• Large sample (>\(L_c\)) and high resolution
• Gouge + fluids

Only partially met by current experiments
Laboratory-derived friction laws

Low resolution experiments (≈ spring+block) record the average stress and slip → macroscopic friction

High resolution experiments are densely instrumented → local friction + rupture nucleation and propagation

\[ S = \text{stress} \]
\[ D = \text{slip} \]

Large scale experiment
Dieterich (1980)
Slip weakening (e.g. Ohnaka) is the main effect during fast dynamic rupture.

*Linear* slip weakening is a simplified model.

Important parameters:

- \( D_c \) = characteristic slip, associated to micro-contact evolution or grain rearrangement.

Without gouge \( D_c \approx 0.1 \) mm.

With gouge \( D_c >10 \) cm

- Strength drop: \( \tau_s - \tau_d \)

Usually a small fraction of normal stress \( \sim 0.1 \sigma \)

- Fracture energy of a linear slip weakening model:

\[
G_c = \frac{1}{2} (\tau_s - \tau_d)D_c
\]
Exponential initiation

Linear slip-weakening:
\[ \Delta \tau = (\tau_s - \tau_d)D / D_c \]
If there is some viscosity in the fault behavior:
\[ \Delta \tau = \eta \dot{D} \]
Equating both:
\[ \dot{D} = sD \]
Hence
\[ D(t) \sim \exp(st) \]
where \( s = (\tau_s - \tau_d) / \eta D_c \)

One form of viscosity is radiation damping, \( \eta = \mu / 2c_s \)
Dynamic Rupture Simulation

Setup:

- Planar fault embedded in homogeneous elastic full space
- Boundary Integral Equation Method (*Dunham, 2005*)

![Dynamic Rupture Simulation Setup Diagram]
Exponential initiation

\[ s_m = 2c_s (\tau_s - \tau_d)/\mu D_c \]

Simulations
Ripperger et al (2007)

\[ s = (\tau_s - \tau_d)/\eta D_c \]

Observations
Tape et al (2013)
Rupture arrest

Rupture “percolation” transition

Ripperger et al (2007)
Rupture arrest

Rupture nucleated at a highly stressed patch.
• Will it stop spontaneously?
• How does the rupture outcome depend on patch size and overstress?

Fault plane

\[ \tau_{\text{nu}} > \tau_0 \]

\[ \tau_0 \]

Nucleation area

\[ S = \frac{\tau_s - \tau_0}{\tau_0 - \tau_d} \]

Runaway ruptures

Stopping ruptures

Background stress

Numerical simulations compared to fracture mechanics predictions (Galis et al, 2014)
Laboratory foreshocks

Rubinstein et al (2007)
Laboratory foreshocks

Kammer et al (2014)

Lab results reproduced by slip-weakening and fracture mechanics models
Rate-and-state friction

Second order effects: logarithmic healing (micro-contact creep) and velocity-weakening

 Phenomenological rate-and-state friction law introduced by Dieterich and Ruina in the early 1980s

Essential ingredients:
- non-linear viscosity
- evolution effect

Most important during slow slip (nucleation and post-seismic)

During fast dynamic rupture, an equivalent $D_c$ can be estimated:

$$D_c \approx 20 \ L$$

$$\mu = \mu^* + a \ln \left( \frac{V}{V^*} \right) + b \ln \left( \frac{V^* \theta}{L} \right)$$

$$\dot{\theta} = 1 - \frac{V \theta}{L}$$

$V = \text{slip velocity}, \ \theta = \text{state variable}$
Integration between dynamic rupture and earthquake cycle modeling

Earthquake Dynamic rupture simulation of The 2011 Tohoku earthquake.

Galvez et al (2014)
Rate-and-state simulations of deep tremor swarms

Tremor swarms result from a cascade of triggering between brittle asperities mediated by creep transients:

Asperity failure

→ propagating creep perturbation
→ loading and failure of next asperity

Quasi-dynamic 3D simulations in collaboration with K. Ariyoshi (Earth Simulator, JAMSTEC, Japan)
Rate-and-state simulations of foreshock swarms

Iquique 2014 foreshock sequence
Slow fronts during nucleation

Kaneko and Ampuero (2011)

Consider a stress concentration \( = F \times \text{length} \) over a background stress drop \( \Delta \tau \).

The static energy release rate as function of distance to the stress concentration \( a \) reaches a minimum at some distance.

This implies a roughly constant rupture speed.

\[
G_0 = \frac{K_0^2}{2 \mu}
\]

\[
\overline{K}_0(a) \approx \sqrt{\pi a} \left( \Delta \tau + \frac{2 F}{\pi a} \right)
\]
Process zone size

In slip-weakening friction:
$$\Lambda \approx \mu D_c / (\tau_s - \tau_d)$$

In rate-and-state friction:
Fast sliding far above steady-state ($V \gg L/\theta$) produces quasi-linear slip-weakening with:
$$D_c \approx L \ln \frac{V}{V^*}$$
$$\tau_s - \tau_d \approx b \sigma \ln \frac{V}{V^*}$$
$$\Lambda \approx \frac{\mu L}{b \sigma} = L_b$$

Rubin and Ampuero (2005)
Nucleation size

Different nucleation regimes depending on $a/b$ (ratio of viscous to weakening effects)

Localized slip at low $a/b$

Expanding slip at high $a/b$

Minimum localization size $= L_b$

Different nucleation sizes

Maximum nucleation size $\sim \left( \frac{b}{b-a} \right)^2 L_b$

Rubin and Ampuero (2005)
Rate-and-state friction and fracture energy

Most important during slow slip (nucleation and post-seismic)

Rate-and-state behaves as slip-weakening during fast dynamic rupture

Equivalent:

\[ D_c = L \ln \left( \frac{V}{V^*} \right) \approx 20L \]

\[ G_c \approx \frac{1}{2} b \sigma L \ln \left( \frac{V}{V^*} \right)^2 \]

Kaneko et al (2008)
Faults operating at low stress

How large is stress drop $\Delta \tau$ compared to strength drop $\tau_s - \tau_d$?

From seismological observations: $\Delta \tau = 1 - 10$ Mpa

From friction and lithostatic overburden:

$$\tau_s - \tau_d = \sigma(\mu_s - \mu_d) = O(100 \, MPa)$$

$\Rightarrow \Delta \tau \ll \tau_s - \tau_d$

Why so small?
Faults operating at low stress

Fault loaded by deep creep

→ stress concentration at the base of the seismogenic zone
Faults operating at low stress

Interseismic stress

Seismogenic depth

Static strength $\tau_s$

Dynamic strength $\tau_d$

Stress drop $\tau_s - \tau_d$
Faults operating at low stress

Interseismic stress

Seismogenic depth W

Average stress drop $\Delta \tau$

Static strength $\tau_s$

Dynamic strength $\tau_d$

Strength drop $\tau_s - \tau_d$

$\tau_s \ll \tau_d$
Faults operating at low stress

Fracture energy balance: \( G_c = \frac{K^2}{2\mu} \sim \frac{\Delta\tau^2 W}{2\mu} \)

\[ \Rightarrow \Delta\tau \sim \sqrt{2\mu G_c / W} \]

Uenishi and Rice’s nucleation size: \( L_c = \frac{\mu D_c}{\tau_s - \tau_d} \)

\[ \Rightarrow \frac{\Delta\tau}{\tau_s - \tau_d} \sim \sqrt{\frac{L_c}{W}} \ll 1 \]
Recurrence time scaling of repeating earthquakes

Recurrence time scaling

\[ T \sim M_0^{0.18} \]

Whereas classical scaling is

\[ T \sim M_0^{1/3} \]

Nadeau and Johnson (1989)
Repeating earthquakes

Model: a circular brittle patch (radius R) embedded in a creeping fault
Repeating earthquakes

Seismogenic zone

Creeping zone

Interseismic slip

Interseismic stress
Recurrence time scaling of repeating earthquakes

Repeating earthquake model: a circular brittle patch (radius $R$) embedded in a creeping fault (steady slip rate $V_{\text{creep}}$)

From fracture mechanics: $\Delta \tau \sim \frac{2\mu G_c}{R}$

From elasticity: $\Delta \tau \sim \mu D / R$

Slip budget: $D = V_{\text{creep}} T$ per event

Seismic moment: $M_0 = \mu \pi R^2 D$

$\Rightarrow T \sim \left( \frac{2G_c}{\mu} \right)^{\frac{2}{5}} \frac{1}{V_{\text{creep}}} M_0^{\frac{1}{5}}$
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