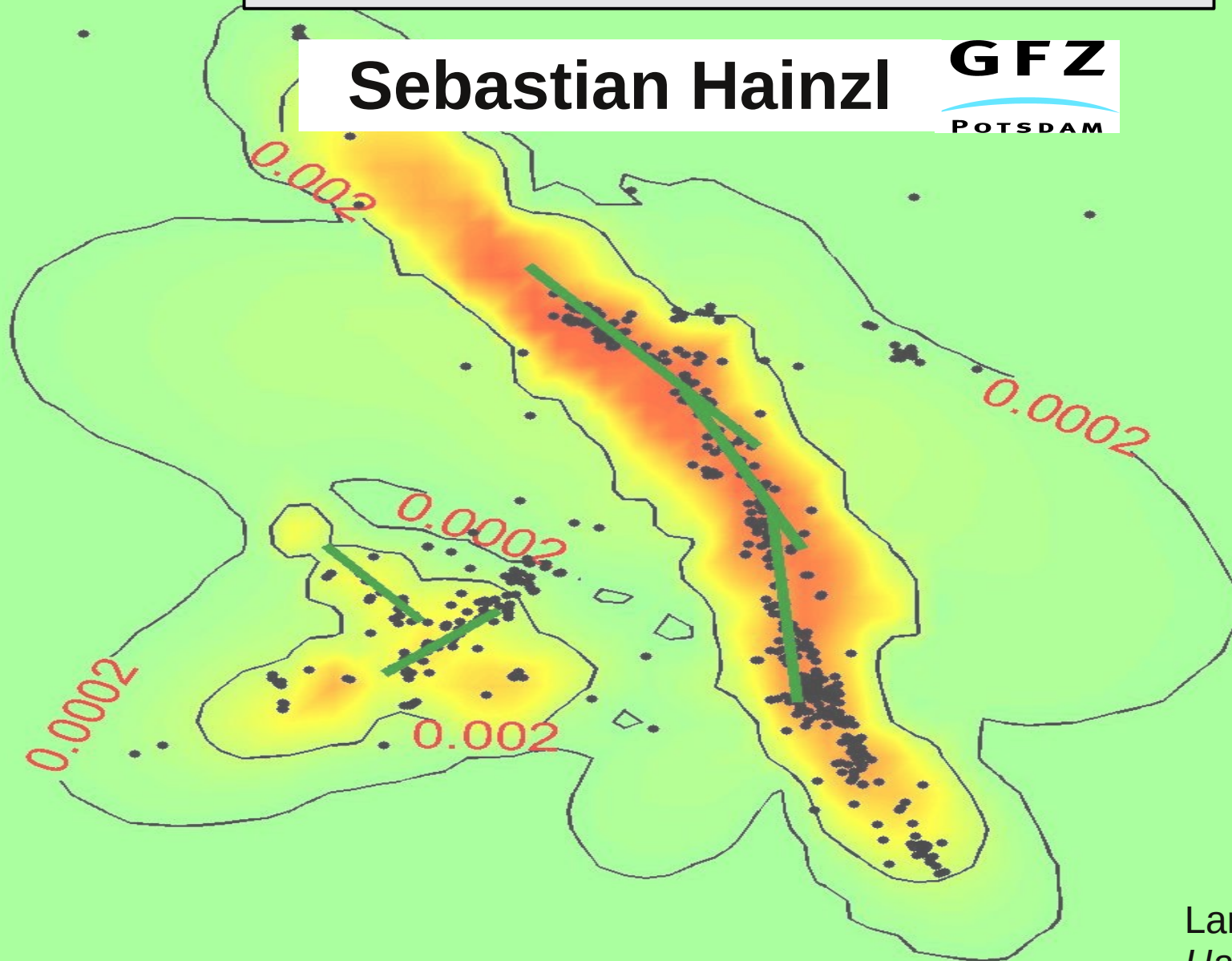


# Seismicity models based on static stress triggering

Sebastian Hainzl

**GFZ**  
POTSDAM



Landers sequence  
Hainzl et al. JGR 2009

... what everybody knows:

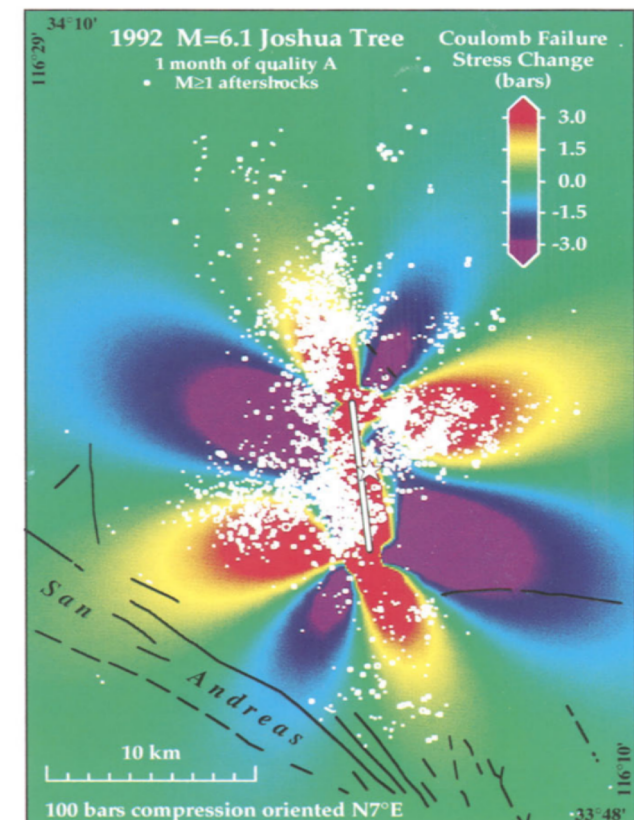
Colorful static stress maps showing generally some correlation between

- aftershock locations and positive stress lobes
- reduced activity and stress shadows

*Stein Nature 1999*



*King et al. BSSA 1994*



**Goal** of this lecture is to understand and discuss quantitative predictions of the static stress triggering model:

- (1) Underlying assumptions
- (2) Predictions of the model
- (3) Potentials & Limitations (due to unknowns/uncertainties) of model applications

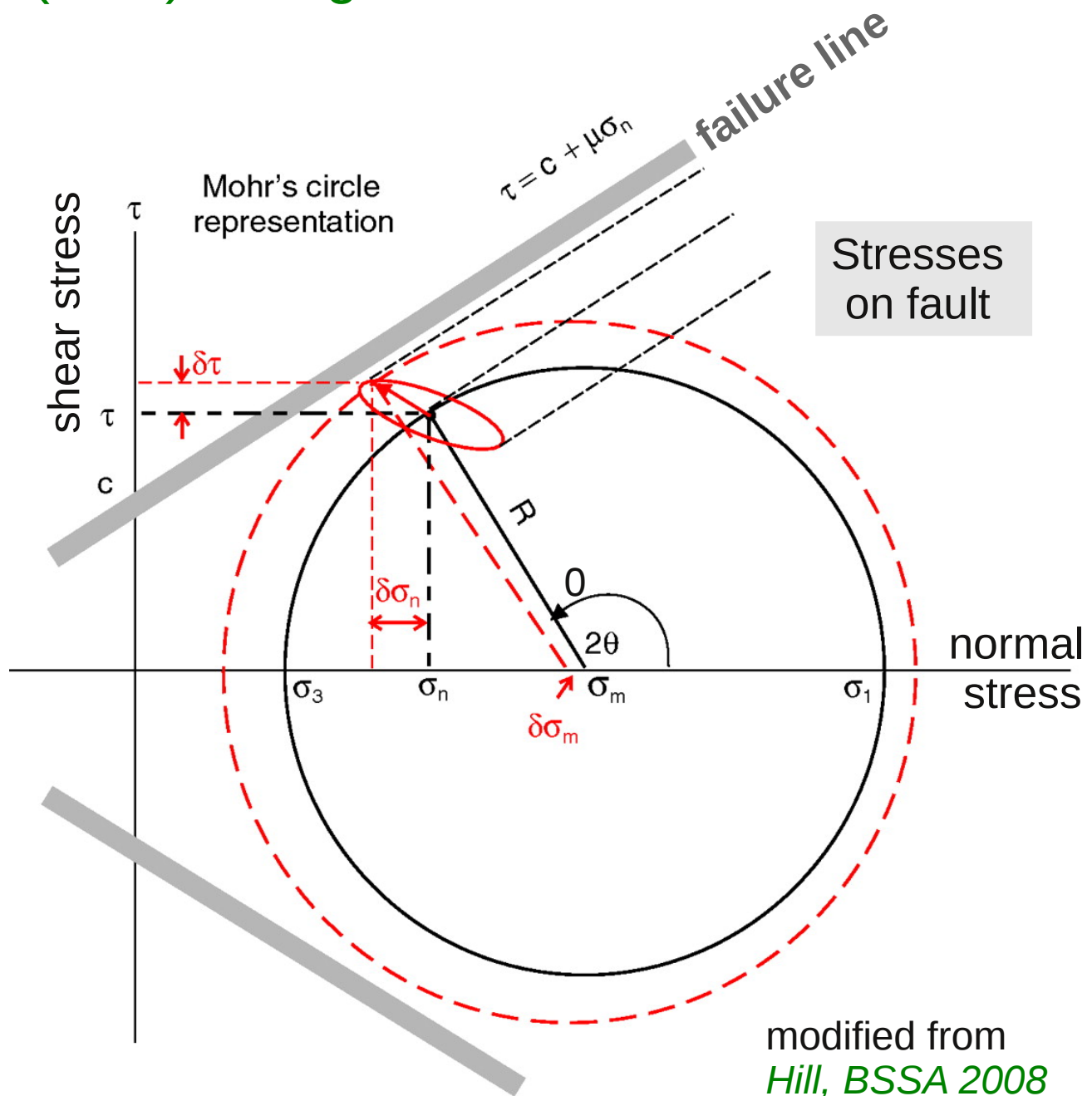
## Outline:

- (1) Neglect timing of earthquakes - A simple clock-advance model:
  - spatial aftershock distribution & decay
  - aftershock productivity
- (2) Time-evolution assuming rate-state-dependent friction:
  - simple considerations
  - examples of model applications

# Coulomb Failure Stress (CFS) change:

$$\Delta CFS = \Delta \tau - \mu \Delta \sigma_n$$

$\tau$  shear stress  
 $\sigma_n$  normal stress  
 (including pore pressure)  
 $\mu$  friction coefficient





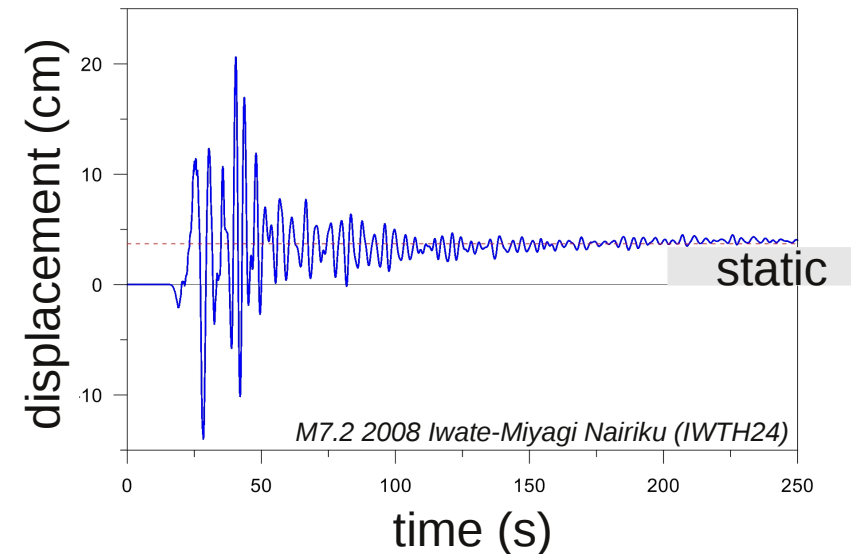
## Origin of stress changes can be:

### (1) **coseismic:**

- dynamic (seismic waves)
- static (permanent)

### (2) **aseismic:**

- pore pressure changes
- continuous or transient creep
- visco-elastic deformation
- dike intrusion
- ...



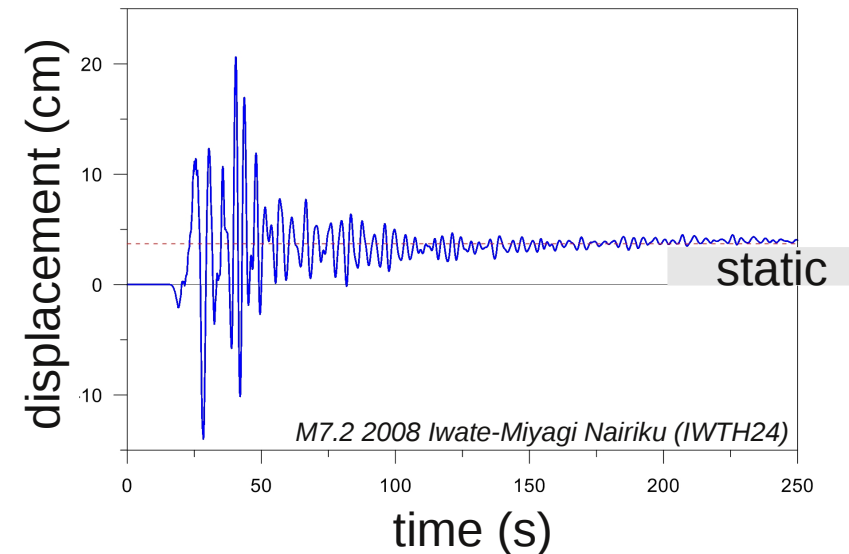
## Origin of stress changes can be:

### (1) coseismic:

- ~~dynamic (seismic waves)~~
- static (permanent)

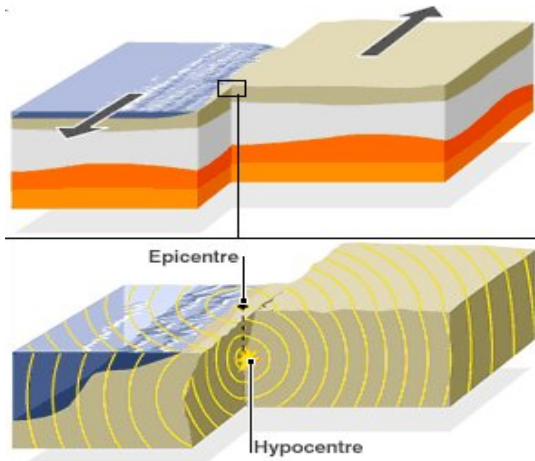
### (2) aseismic:

- pore pressure changes
- continuous or transient creep
- visco-elastic deformation
- dike intrusion
- ...



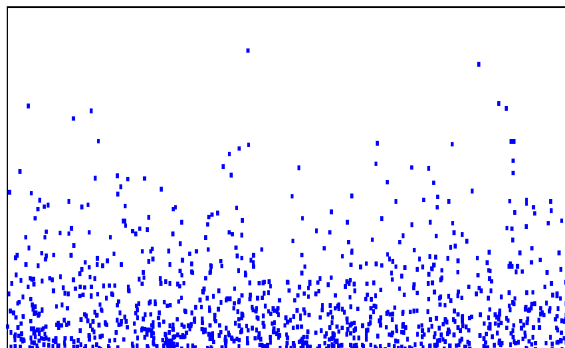
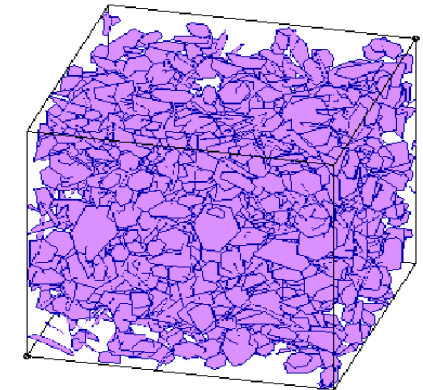
this talk

## Let's start with simplest assumptions:



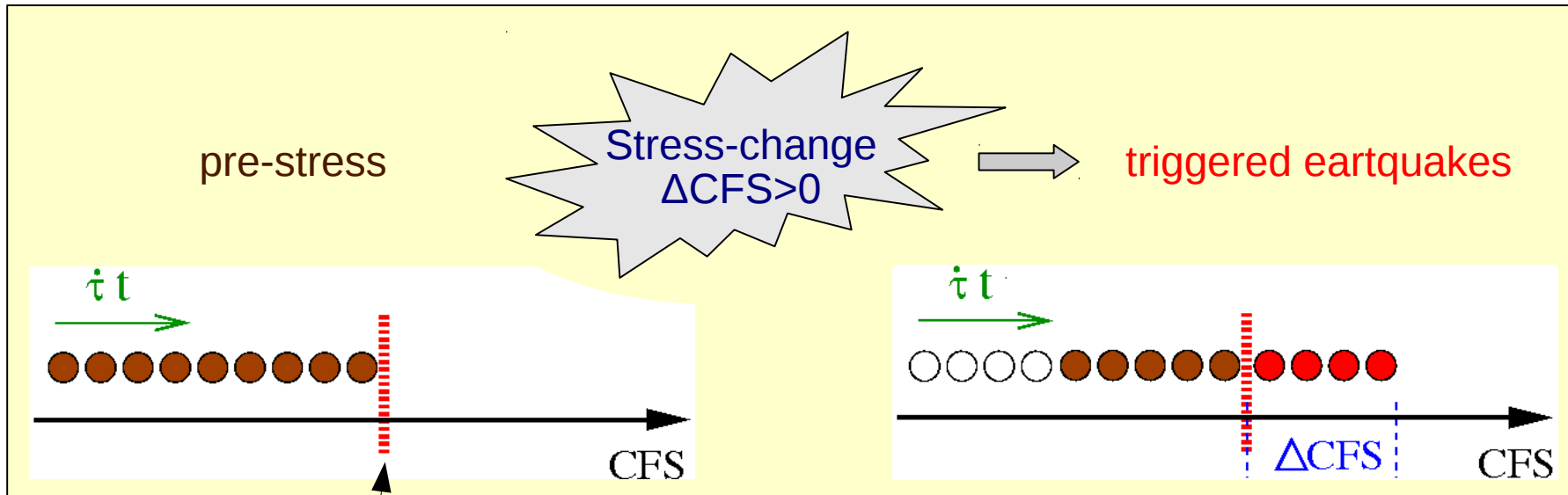
(1) Constant stress build-up due to tectonic loading

(2) Existence of a population of faults where earthquake can occur



(3) **Without** any stress disturbance:  
Earthquake rate is constant  
→ Poisson model  
(assumption of seismic hazard assessment)

Hainzl et al. JGR 2010



Critical stress at which earthquake nucleation starts

**Clock advance** - time needed to achieve  $\Delta CFS$  by tectonic loading:  $\Delta t = \Delta CFS / \dot{\tau}$

Number of triggered events:  $N_a = r \Delta t = (r / \dot{\tau}) \Delta CFS$  if  $\Delta CFS > 0$   
else  $N_a = 0$



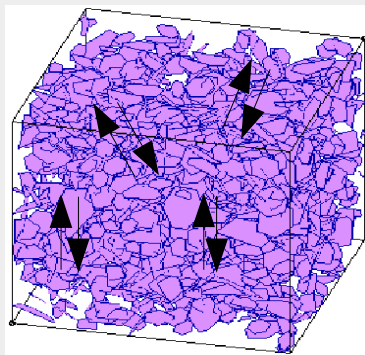


Number of triggered events  $\sim \Delta\text{CFS}$  (if  $\Delta\text{CFS} > 0$ )

Proportionality factor:  $c = \frac{r}{\dot{\tau}} \approx \frac{V}{\overline{M}_0}$

*Ratio between seismogenic volume and the average seismic moment per earthquake*

*Kostrov 1974:*



$$\begin{aligned} \dot{\tau} &= 2G \dot{\epsilon}_{ij} = \lim_{\Delta t \rightarrow \infty} \frac{1}{V \Delta t} \sum_{k=1}^N M_{0ij}^k \\ &= \lim_{\Delta t \rightarrow \infty} \frac{N}{V \Delta t} \overline{M}_{0ij} = \frac{r}{V} \overline{M}_{0ij} \\ &\approx \frac{r}{V} \overline{M}_0 \end{aligned}$$

Assumption: Similar earthquake mechanisms

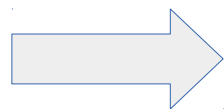
$$N_a = (V / \bar{M}_0) \Delta CFS$$

For Gutenberg-Richter distributed earthquakes within  $[M_{min}, M_{max}]$ :

$$pdf(M) = \ln(10) b \frac{10^{-b(M - M_{min})}}{1 - 10^{-b(M_{max} - M_{min})}}$$

Average seismic moment per earthquake (NO time average!):

$$\begin{aligned} \bar{M}_0 &= \int_{M_{min}}^{M_{max}} pdf(M) 10^{9.1 + 1.5M} dM \\ &= 10^{9.1 + 1.5M_{min}} \frac{b}{1.5 - b} \frac{10^{(1.5 - b)(M_{max} - M_{min})} - 1}{1 - 10^{-b(M_{max} - M_{min})}} \end{aligned}$$

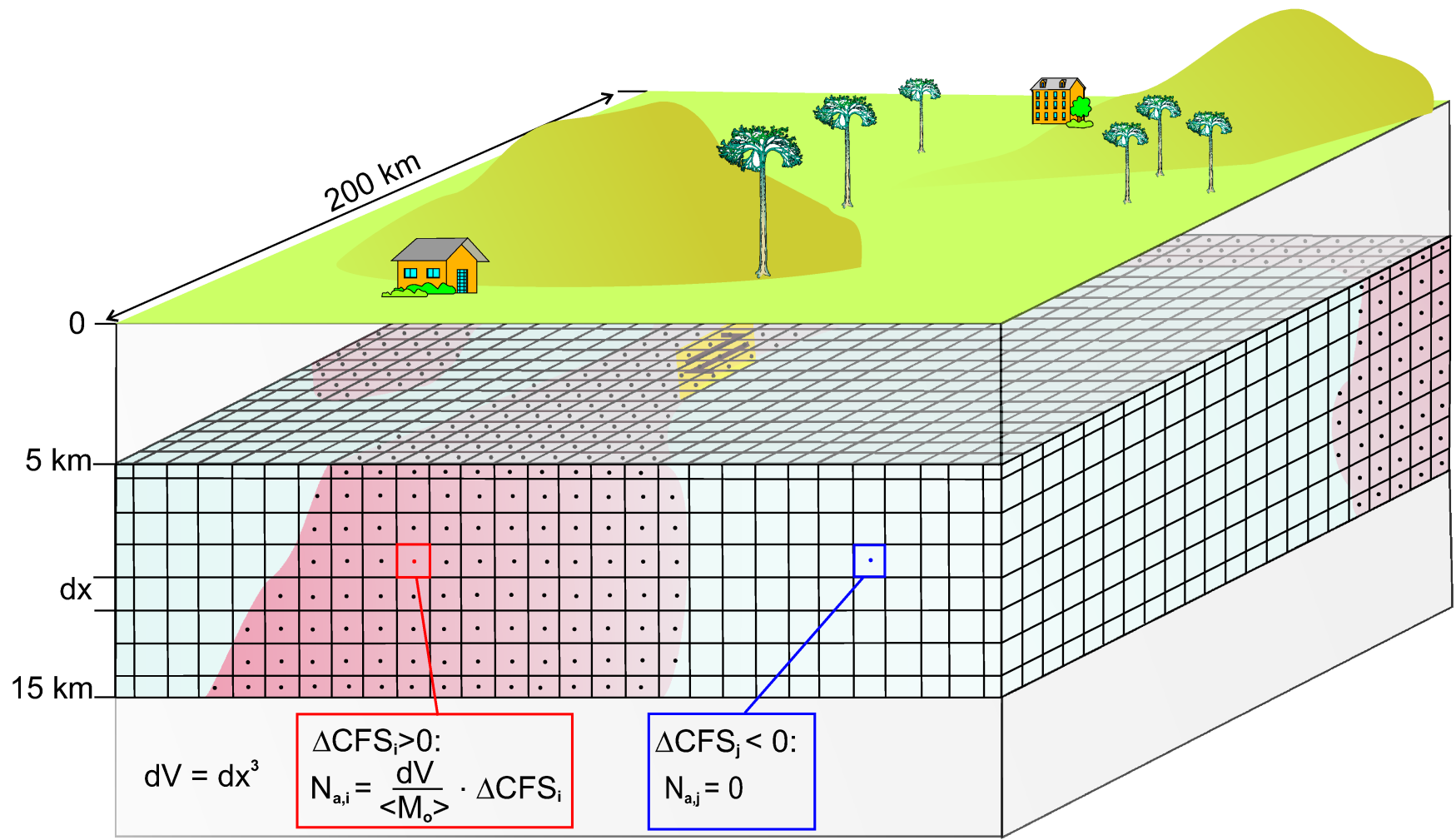


Proportionality factor  $c = V / \bar{M}_0$   
can be calculated for given  $b$  and  $M_{max}$  ( $M_{min}$ )



$$N_a(x) \sim \Delta\text{CFS}(x)$$

(for  $\Delta\text{CFS} > 0$ )

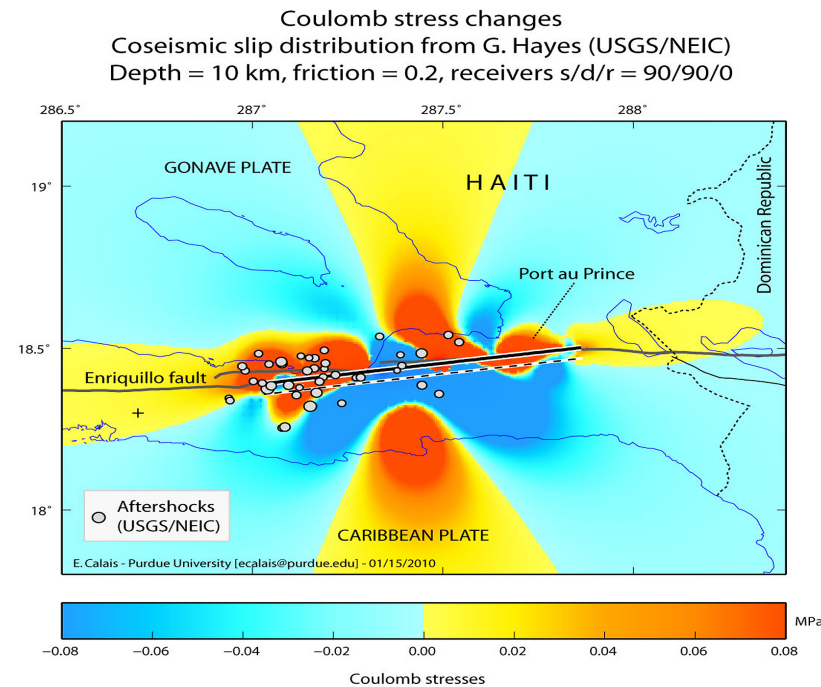


## Model Prediction:

No earthquakes in regions with  $\Delta\text{CFS} < 0$  (stress shadows)

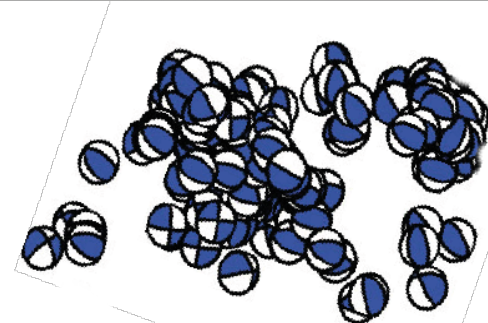
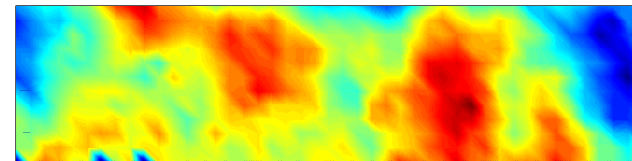
## Observation:

A significant fraction of earthquakes occur in stress shadows ...  
is static stress triggering not working?



## Not necessarily because of:

1. uncertainty of slip inversions
2. unresolvable small scale slip
3. variable receiver mechanisms
4. secondary stress changes



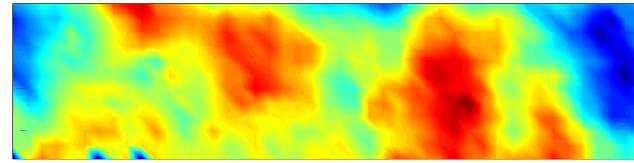


# Small scale slip variability

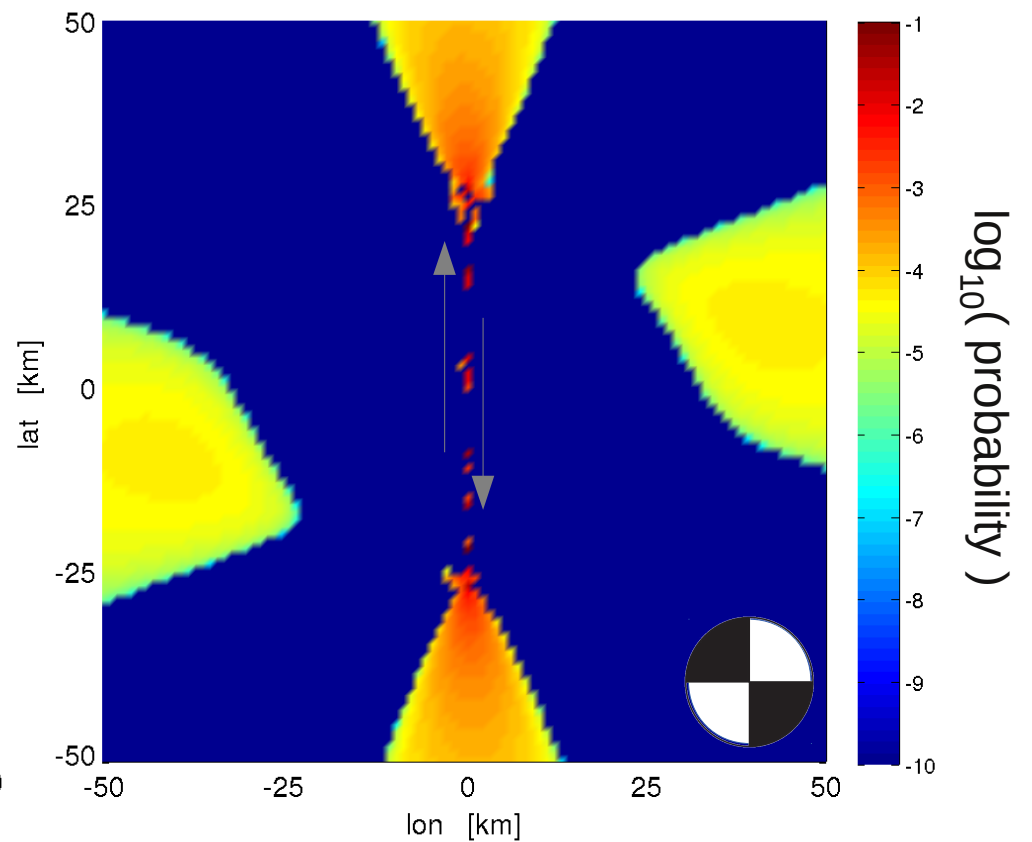
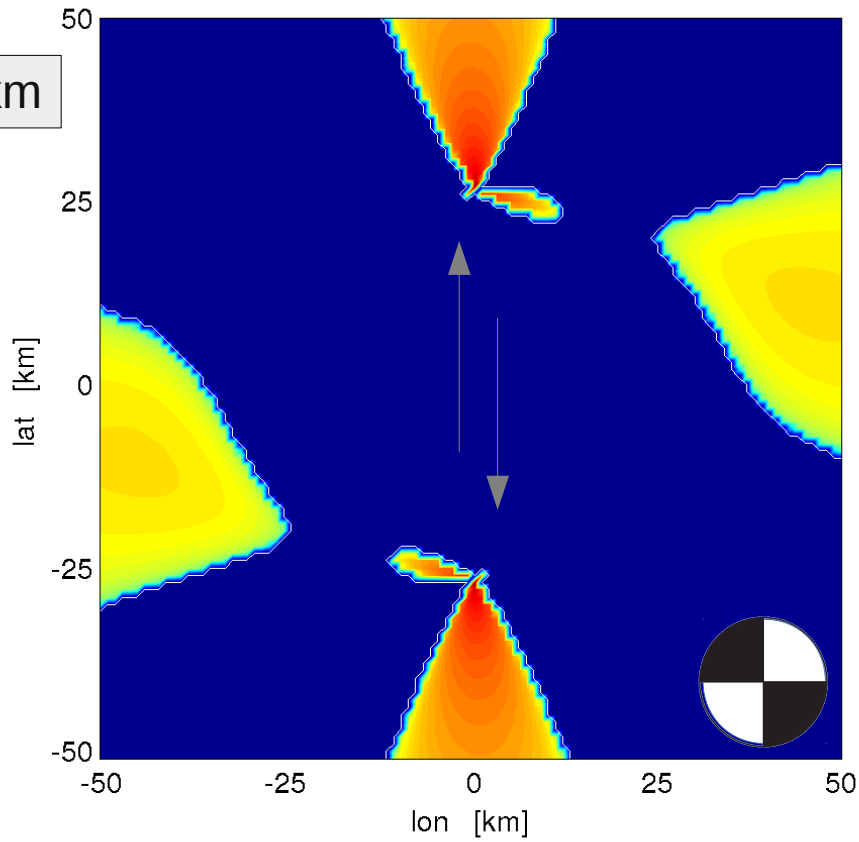
uniform



fractal



z=7km

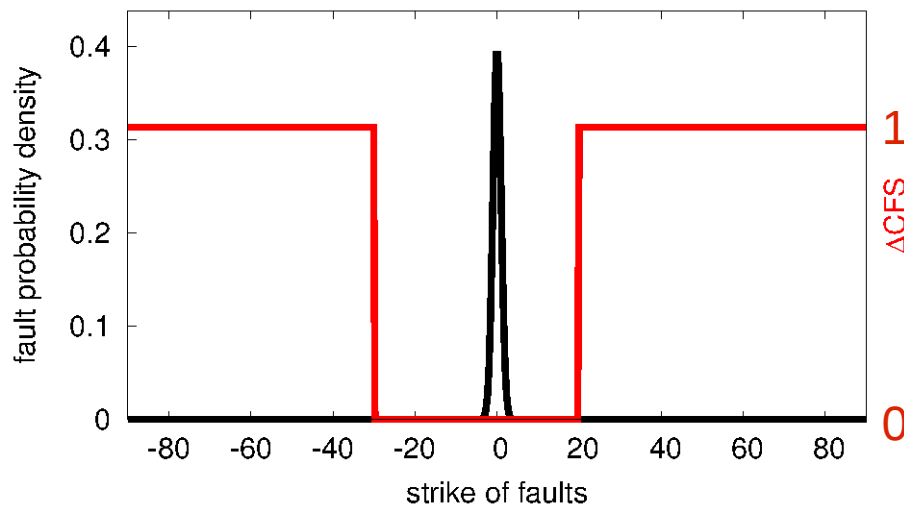


→ can explain on-fault activation

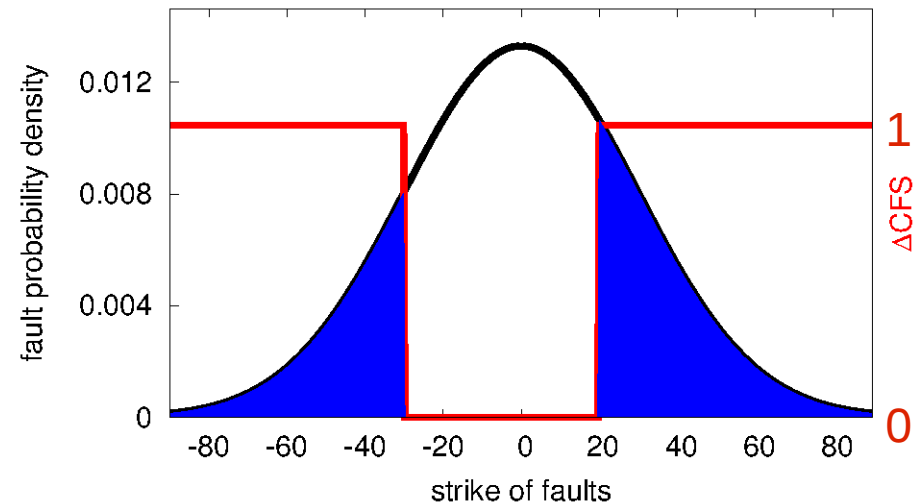
## Variable receiver mechanisms

Faults with different orientations always exists

Examples:



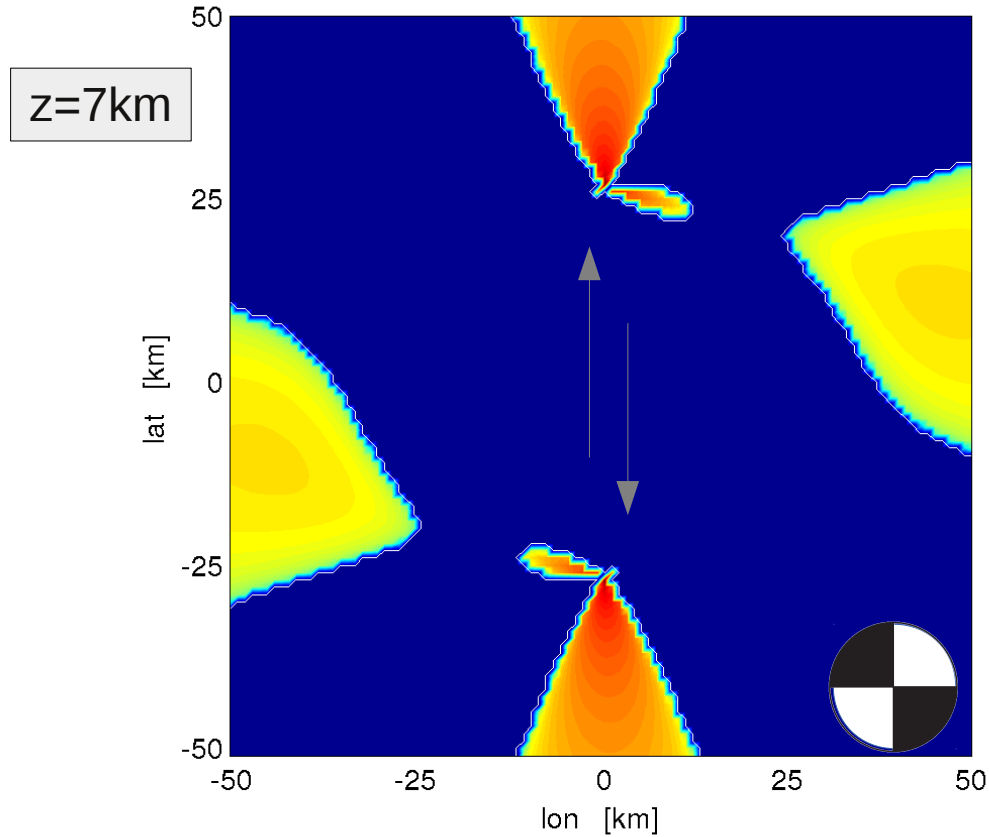
“unique” mechanism:  $\Delta CFS < 0$   
→ no aftershocks



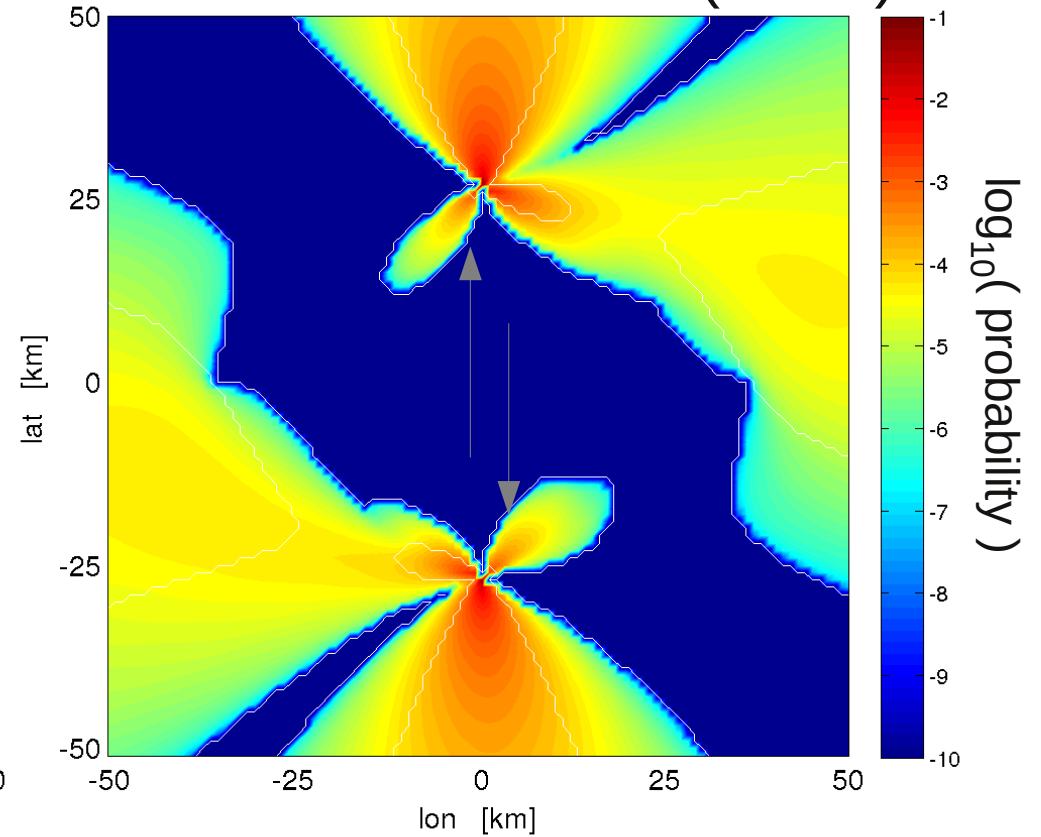
mean mechanism:  $\Delta CFS < 0$   
but some faults have  $\Delta CFS > 0$   
→ aftershocks ~ blue area

# Variable receiver mechanisms

## fixed mechanism

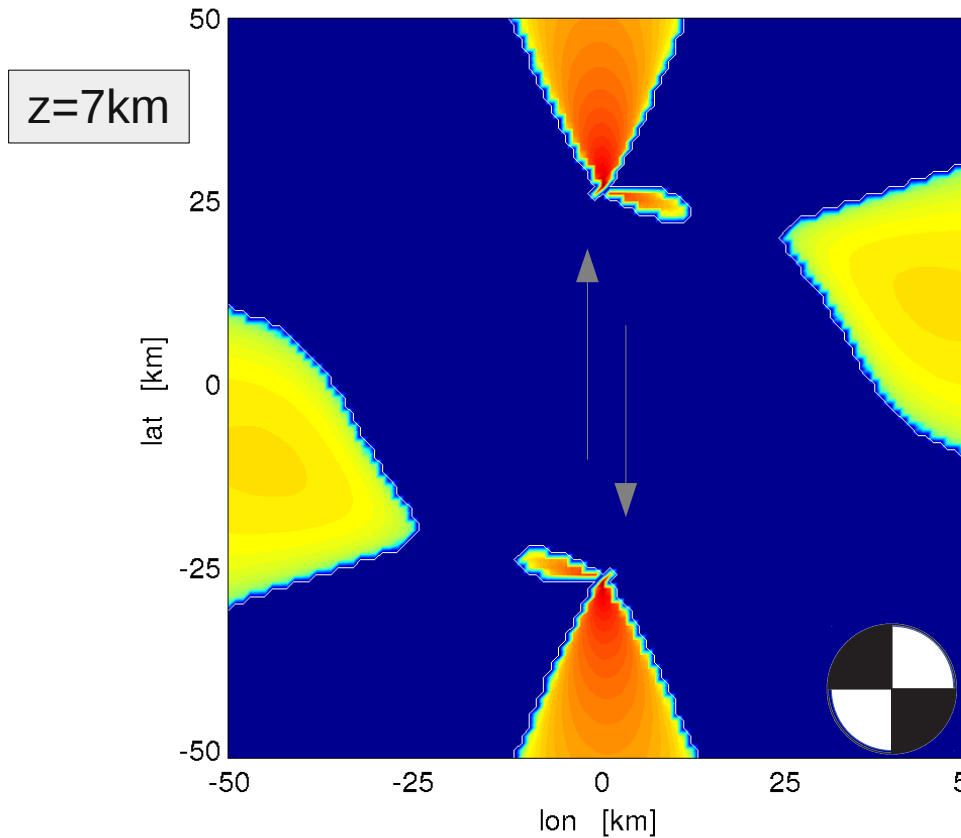


## variable mechanisms ( $\pm 10^\circ$ )

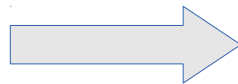
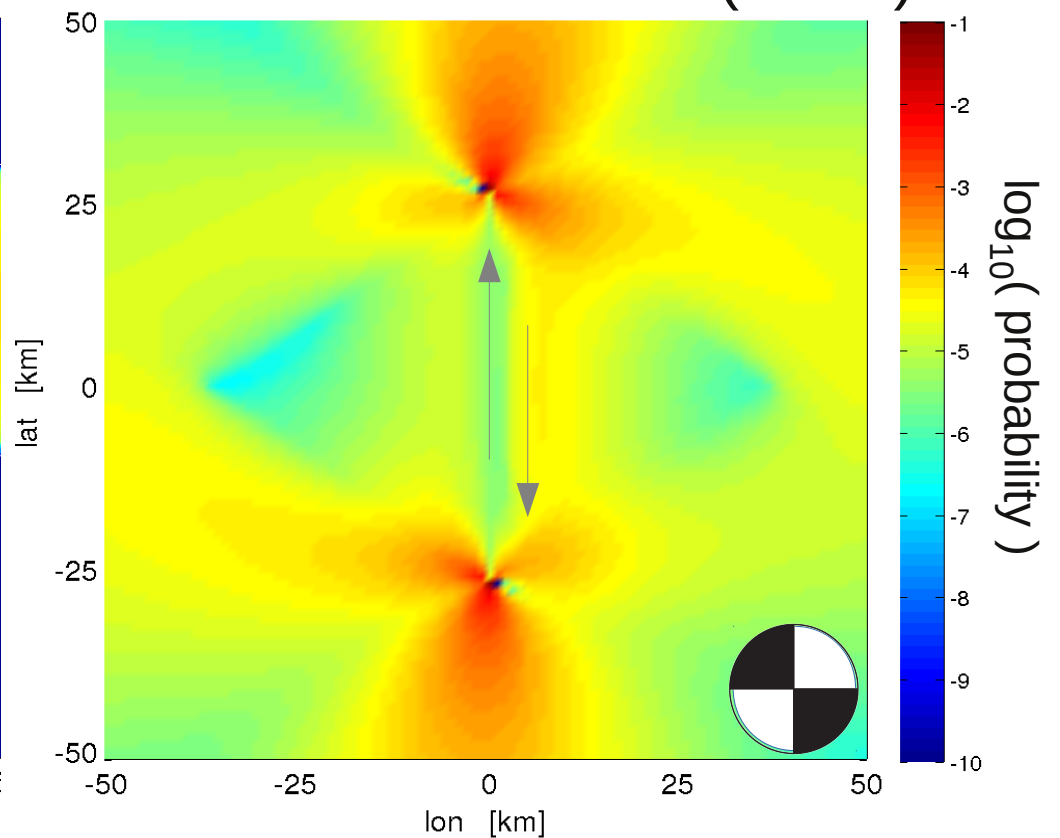


# Variable receiver mechanisms

fixed mechanism



variable mechanisms (+-30°)



NO absolute stress shadows,  
only regions with reduced activation!

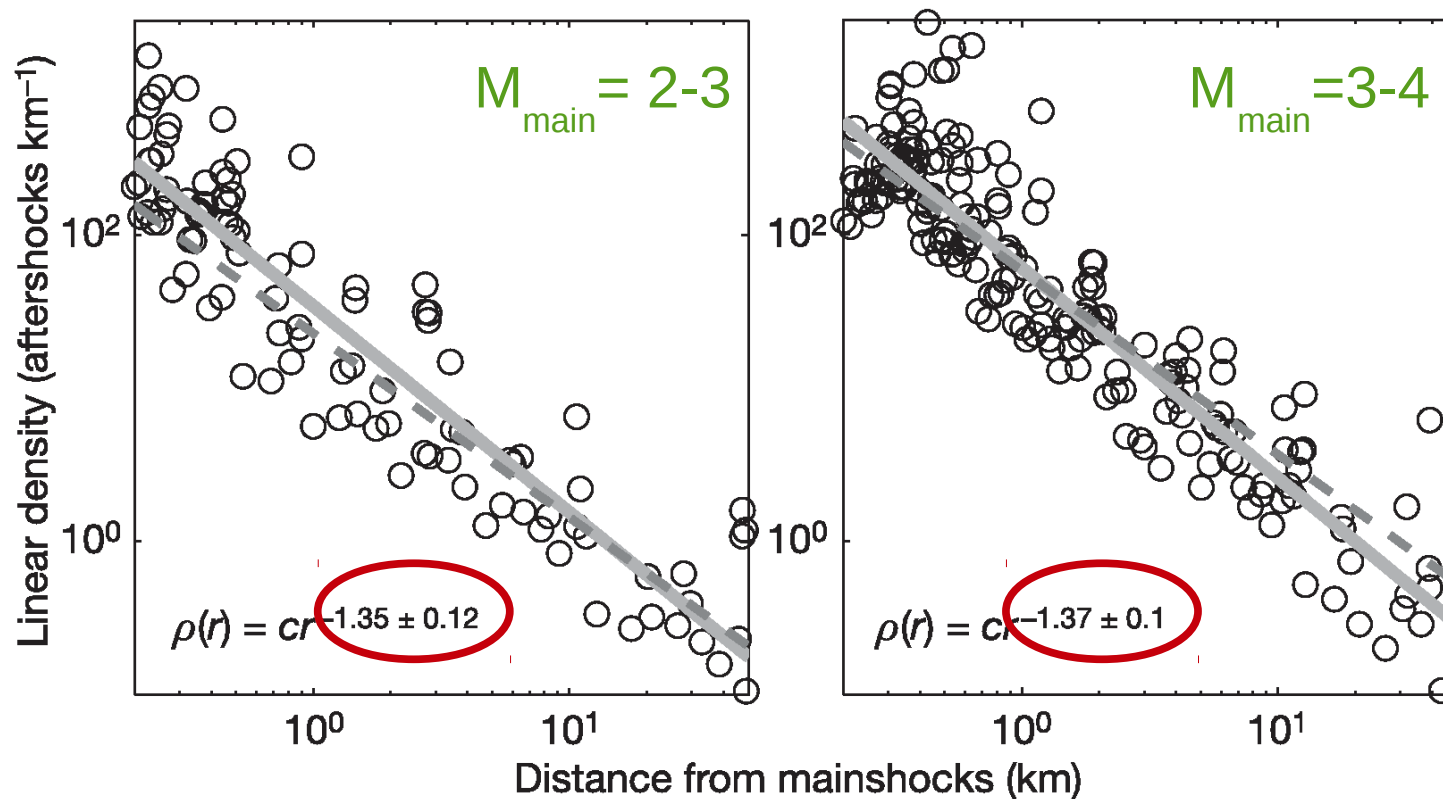


# Distance decay

**Theory** (homogeneous elastic full-space):

Far-field: dynamic stress  $\sim 1/r^2$   
static stress  $\sim 1/r^3$

*Felzer & Brodsky, Nature 2006:*



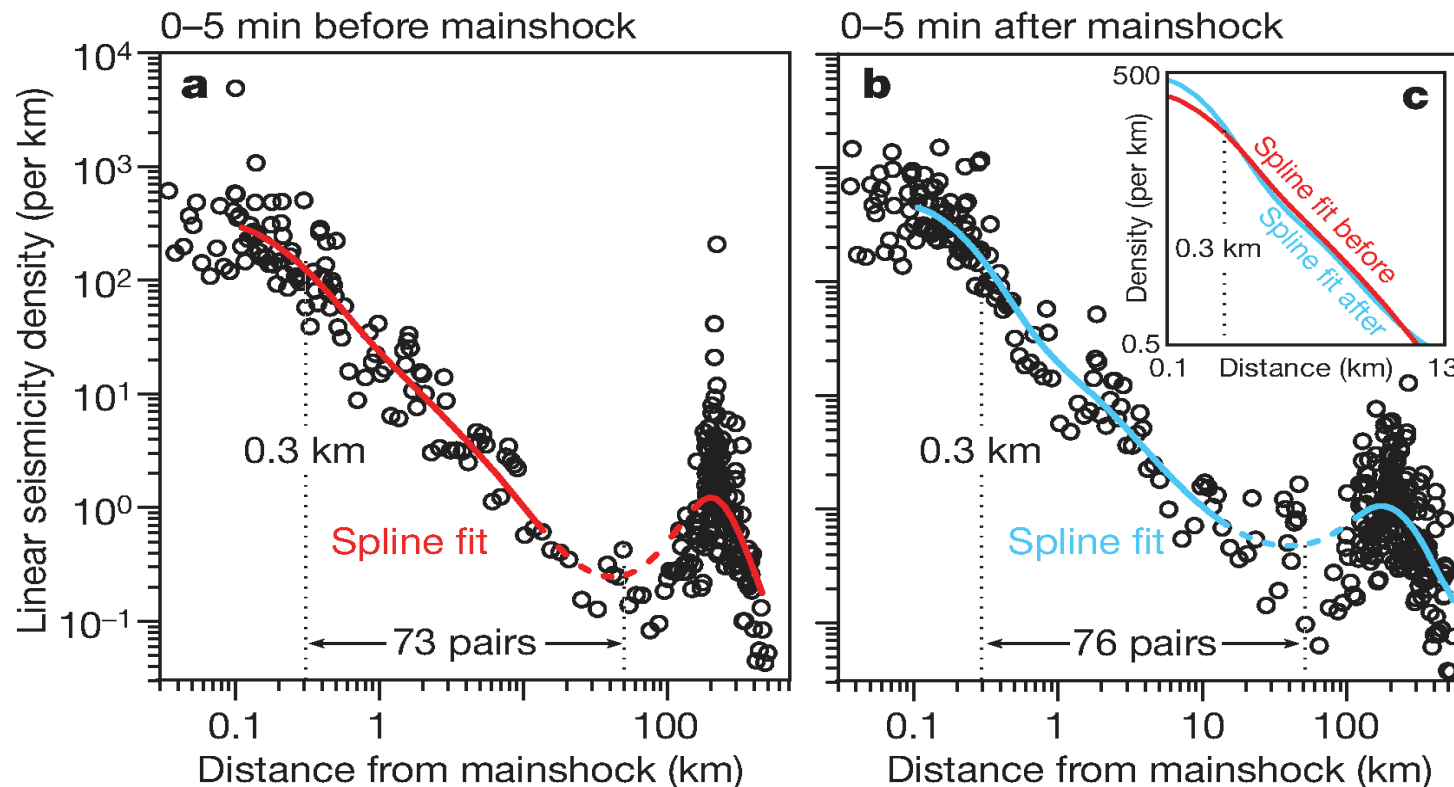
Static stress cannot be the driving force

# Distance decay

Theory for homogeneous elastic full-space:

Far-field: dynamic stress  $\sim 1/r^2$   
static stress  $\sim 1/r^3$

*Richards-Dinger et al., Nature 2010:*

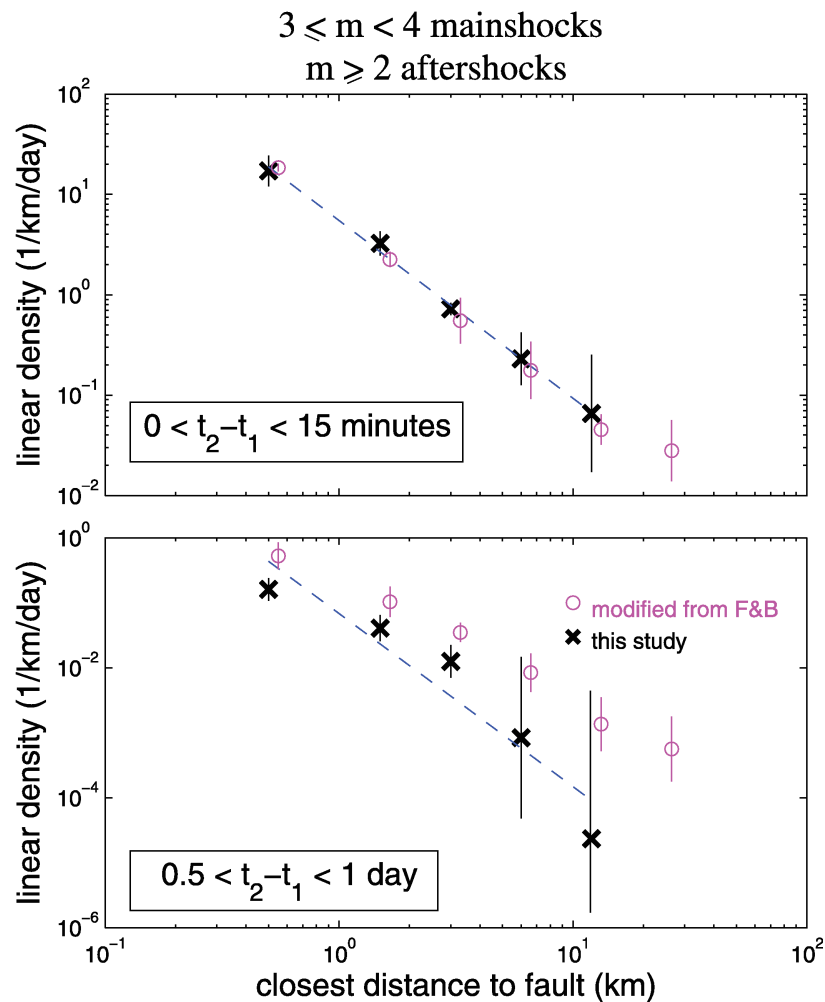


improper selection of “aftershocks” does not allow this conclusion

# Distance decay

Theory for homogeneous elastic full-space:

Far-field: dynamic stress  $\sim 1/r^2$   
static stress  $\sim 1/r^3$



*Marsan & Lengliné, JGR 2010:*

... with an improved statistical attempt to isolate earthquakes causally related to the mainshock:

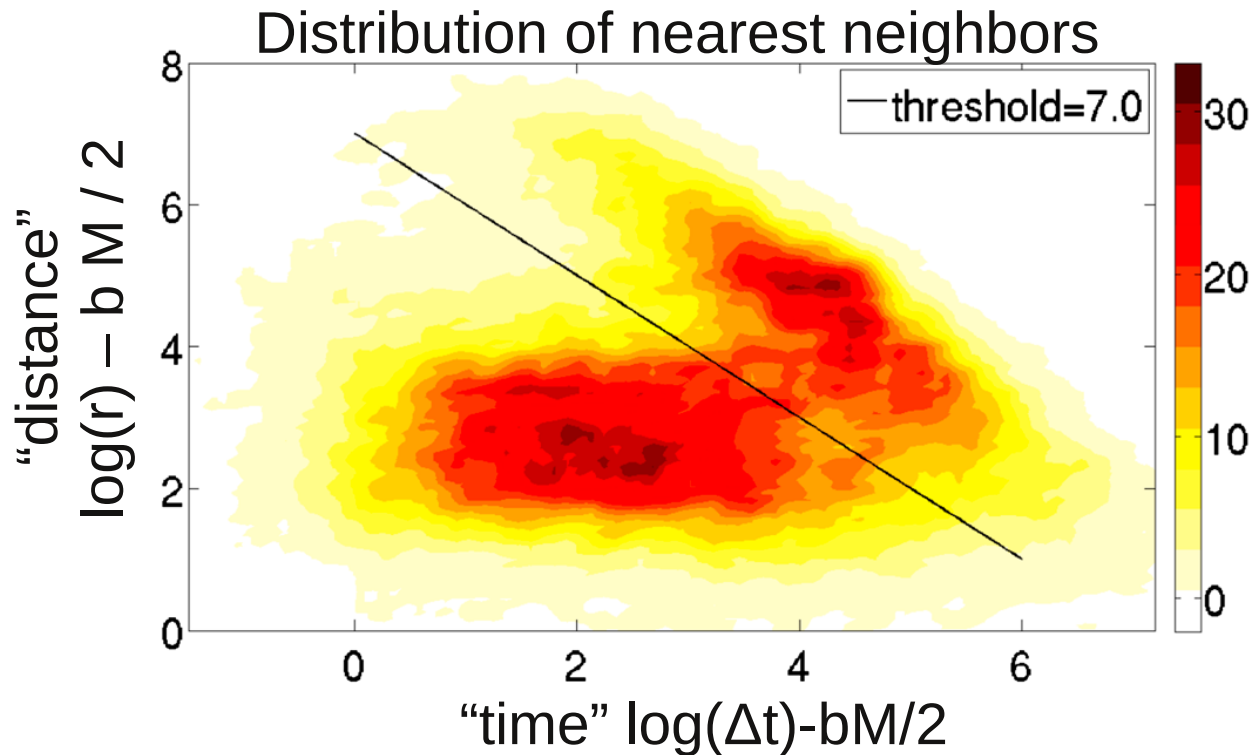
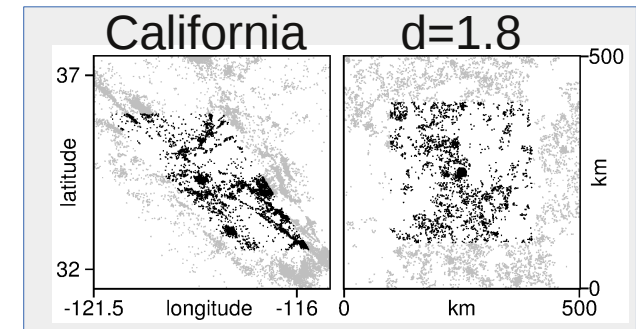
Exponent = 1.7-2.1

*Moradpour et al. JGR 2014*

Separation based on the smallest space-time distance

$$n_{ij} \sim \Delta t r^d 10^{-bM}$$

between an event and all preceding events, where  $d$  is the assumed fractal dimension.

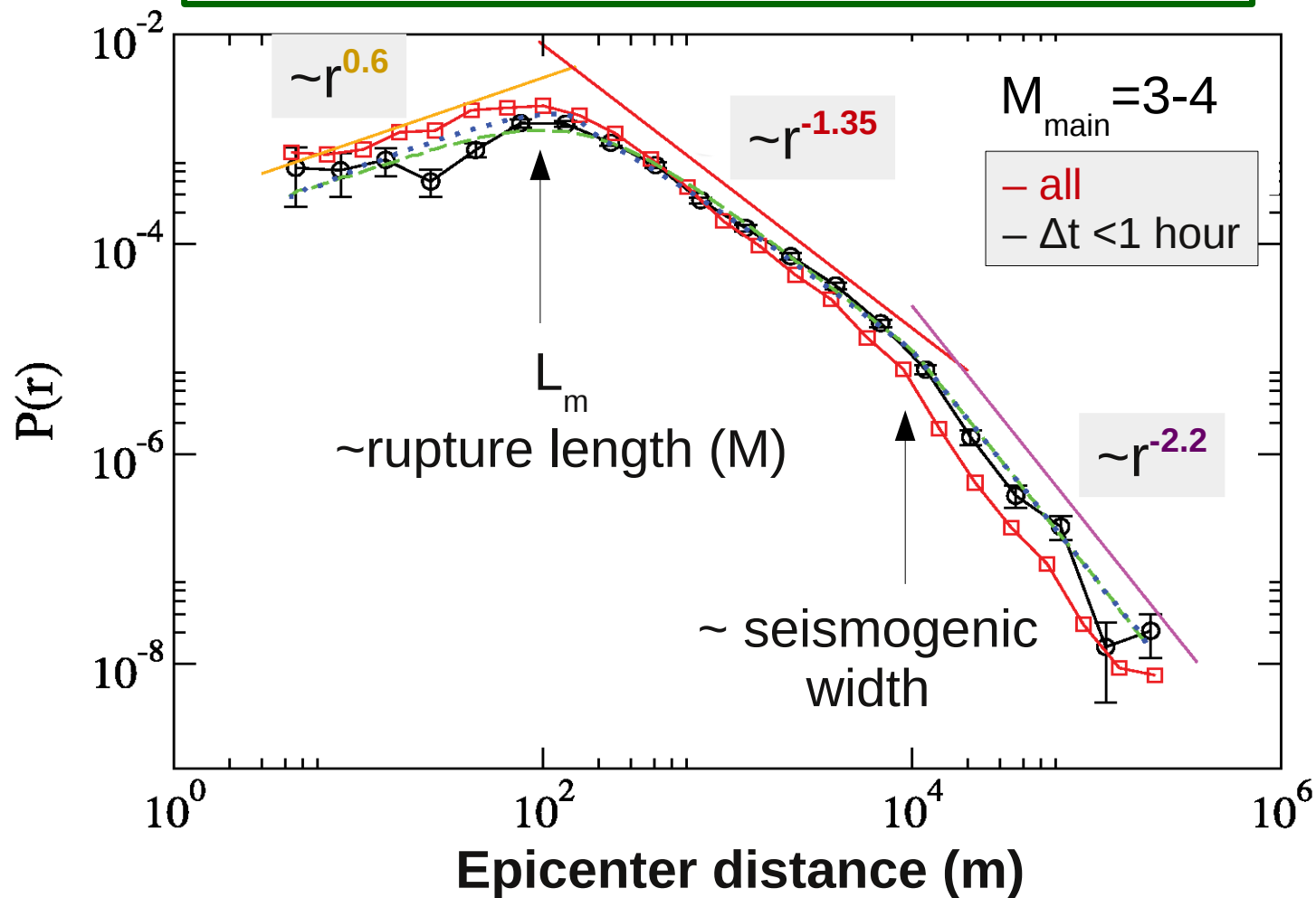


Method based on  
*Biasi & Paczuski 2004*  
*Zaliapin et al. 2008*



$$P_m(r) = \begin{cases} \alpha \frac{qr^\gamma}{L_m^{\gamma+1} \left( \frac{r^{\gamma+1}}{L_m^{\gamma+1}} + 1 \right)^{1 + \frac{q}{\gamma+1}}} & \text{if } r < 10\text{km,} \\ \beta \frac{dr^\gamma}{L_m^{\gamma+1} \left( \frac{r^{\gamma+1}}{L_m^{\gamma+1}} + 1 \right)^{1 + \frac{d}{\gamma+1}}} & \text{if } r > 10\text{km.} \end{cases}$$

$$\begin{aligned} \gamma &= 0.6 \\ q &= 0.35 \\ d &= 1.2 \end{aligned}$$



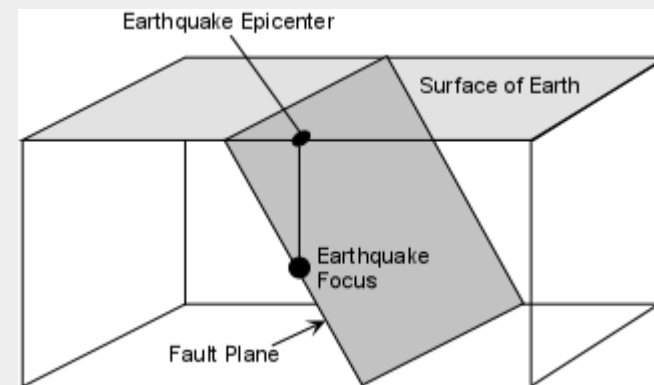
**3 scaling regimes**

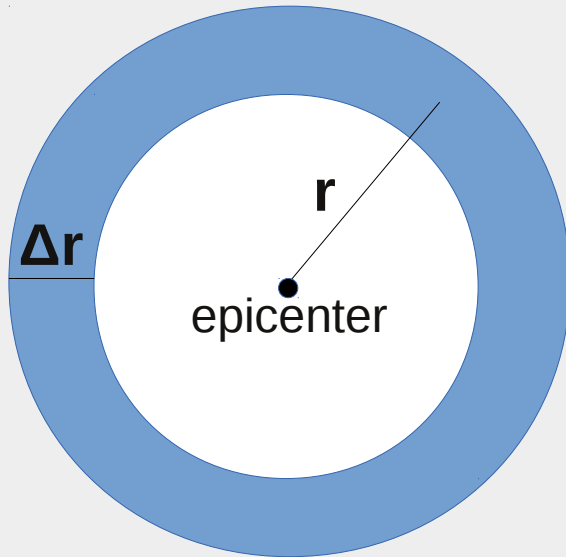
Southern California

## ... from empirical observations back to static stress-triggering:

### Analysis for synthetic mainshock ruptures:

- select magnitude  $M$
- uniform (or fractal) slip
- empirical relations between  $M$  and slip/area
- epicenter randomly chosen within rupture area
- effective friction coefficient:  $\mu = 0.5$
- elastic half-space (or layered half-space)





$$A \sim (r + \Delta r/2)^d - (r - \Delta r/2)^d$$

A: seismogenic area at distance  $r$

$d$ : fractal fault dimension (uniform:  $d=2$ )

Linear aftershock density:

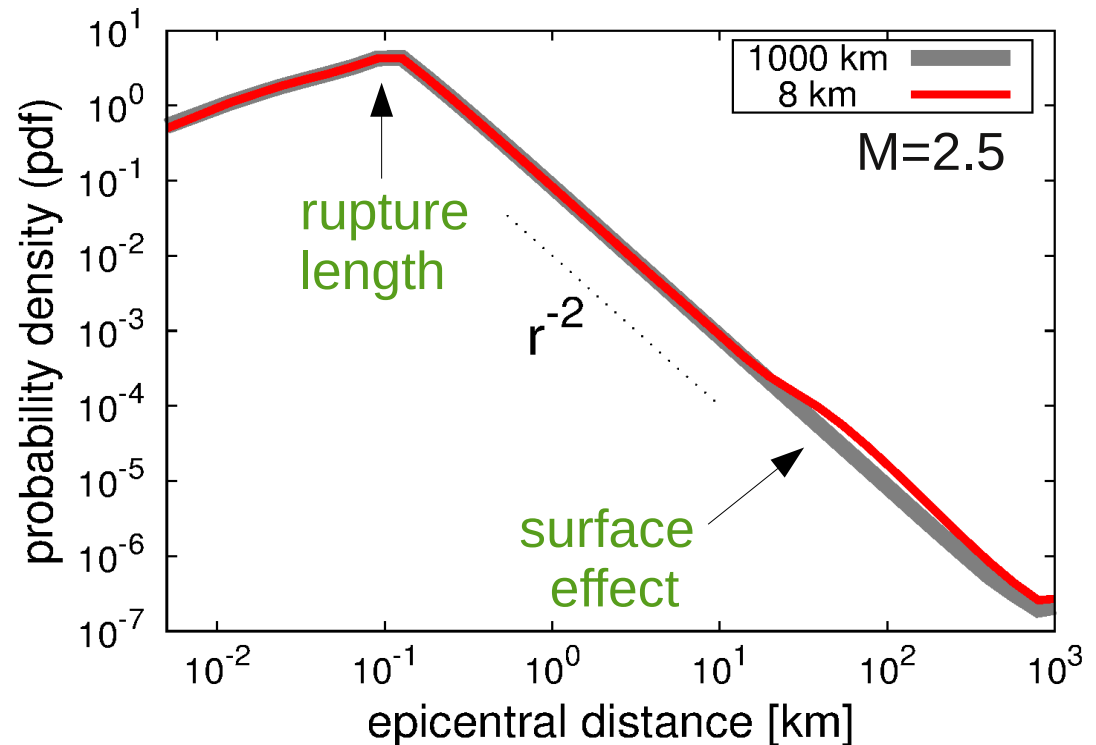
$$P(r) \sim \frac{\Delta \text{CFS}(r) H(\Delta \text{CFS})}{A / \Delta r}$$

Aftershocks in the hypocenter depth layer:

Far field:  $\Delta \text{CFS} \sim r^{-3}$

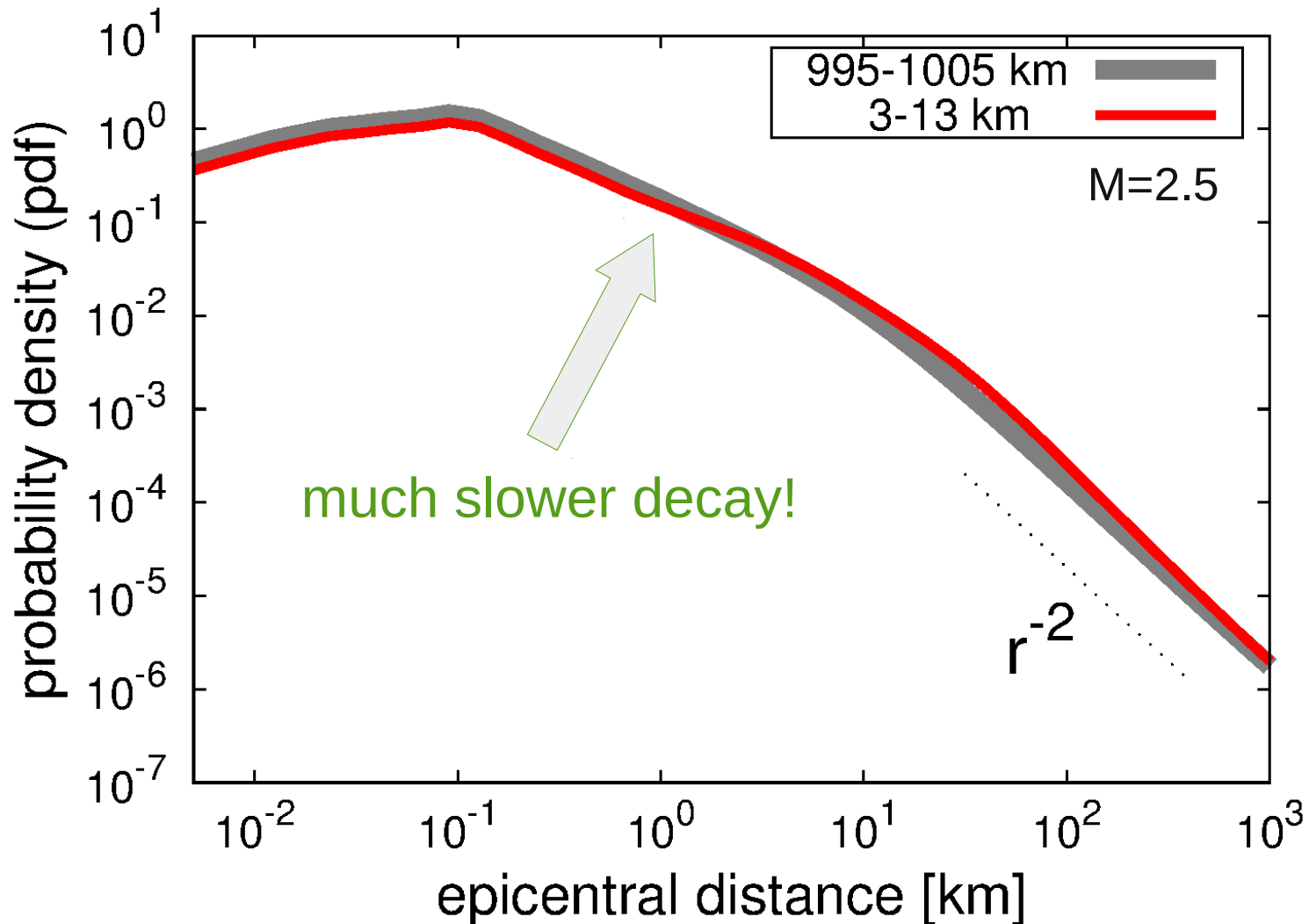
$$A \sim r$$

$$\rightarrow N_a \sim r^{-2}$$

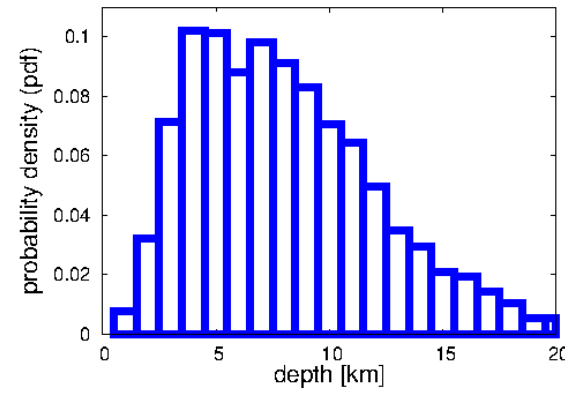


... but aftershocks occur not only at the hypocenter depth layer:

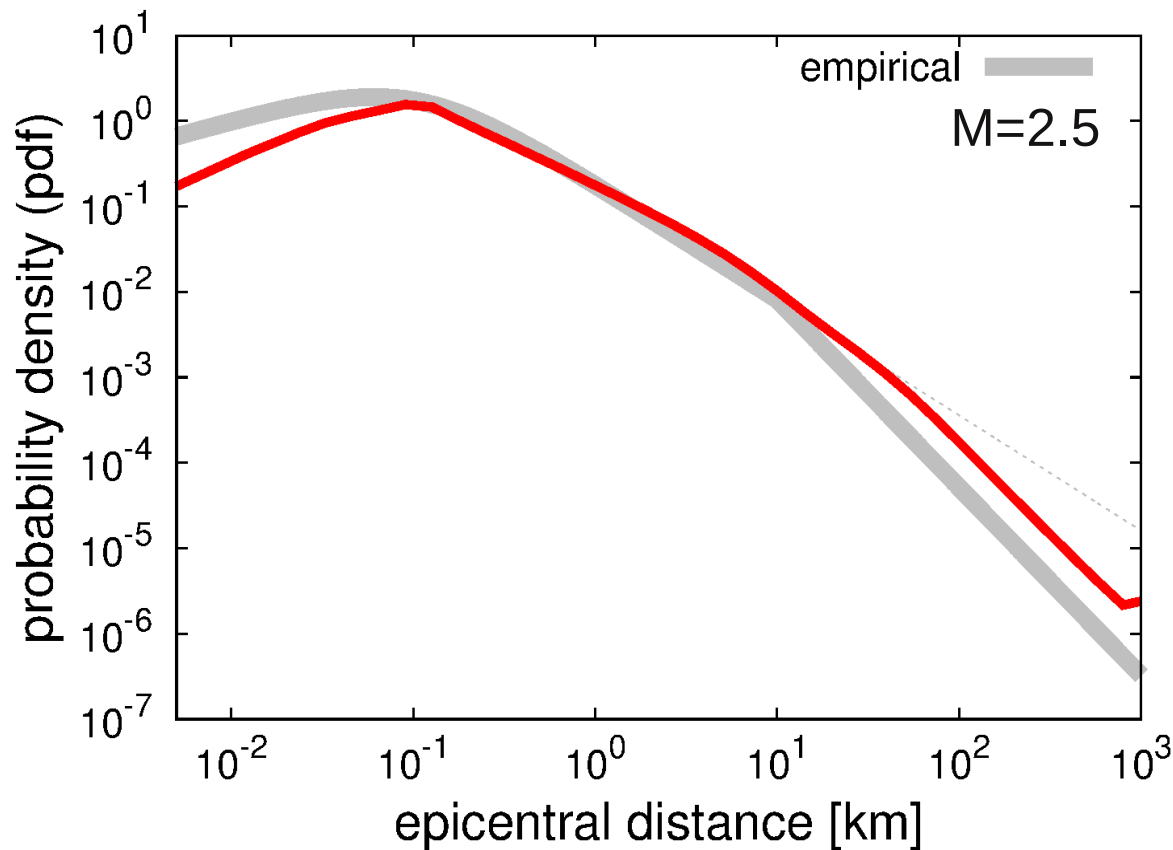
Integration over depth interval



Adaption to Southern California seismicity by random selection of depth & focal mechanism

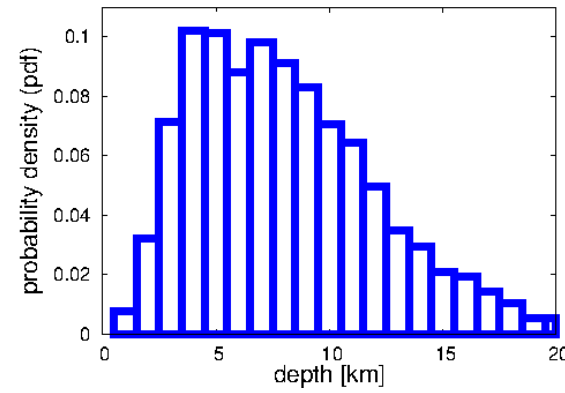


*Yang, Hauksson & Shearer (2012)*

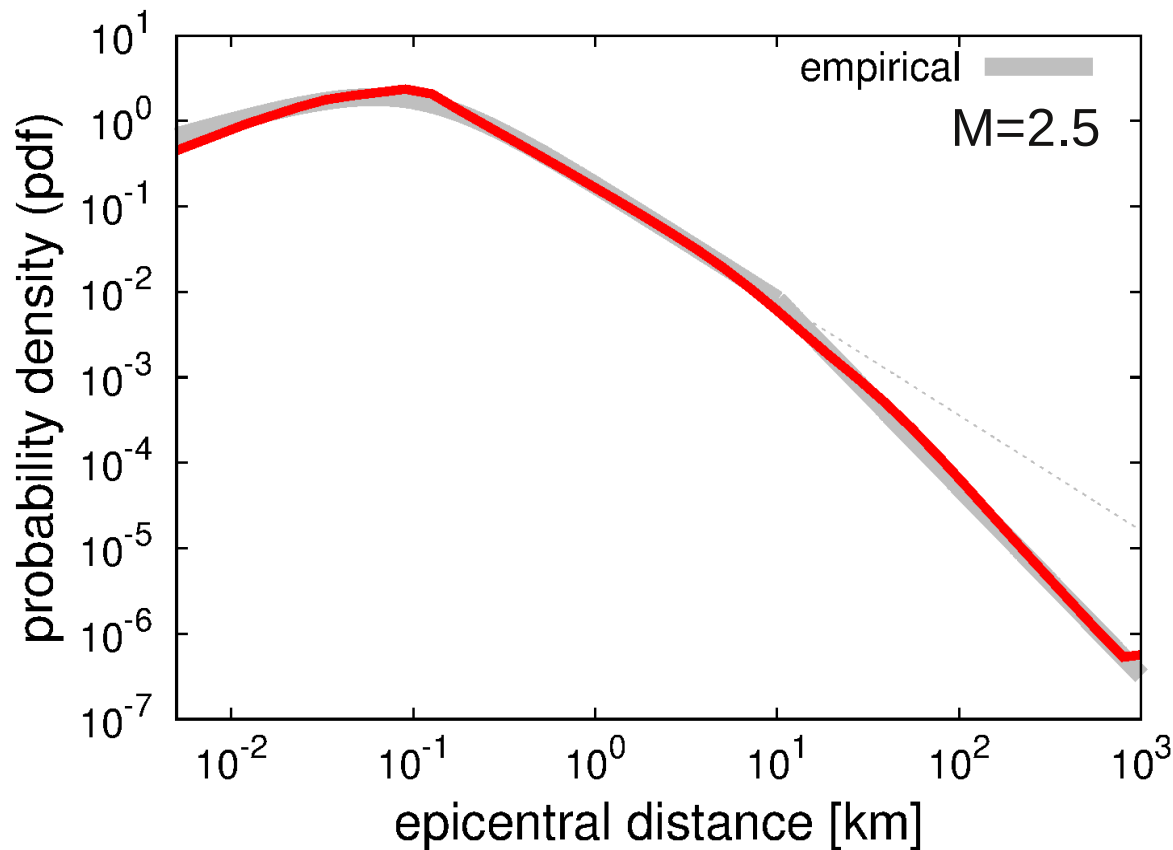


adapted model  
forecast  
**d=2, uniform**

Adaption to Southern California seismicity by random selection of depth & focal mechanism

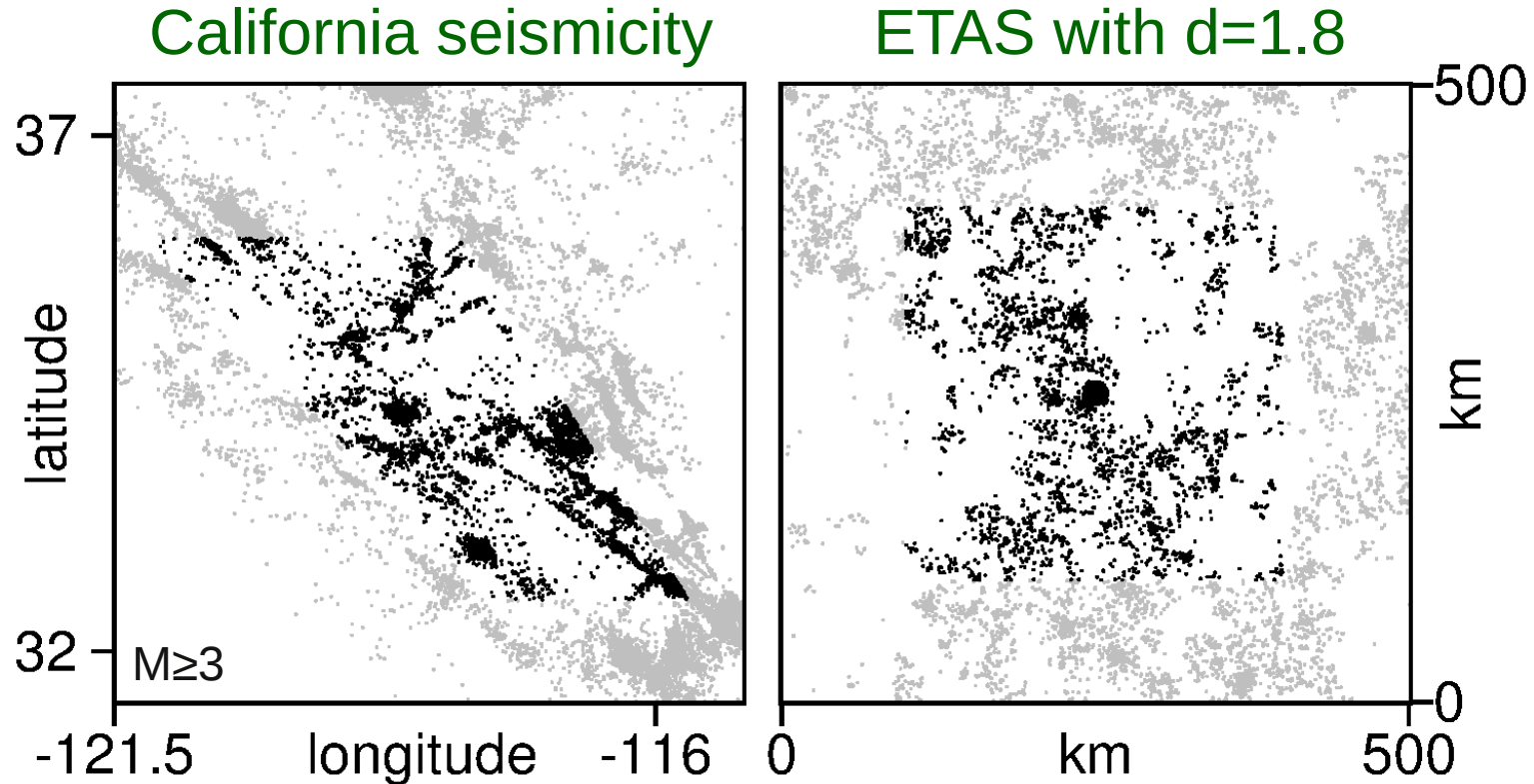


*Yang, Hauksson & Shearer (2012)*



adapted model  
forecast  
**d=1.8, fractal**

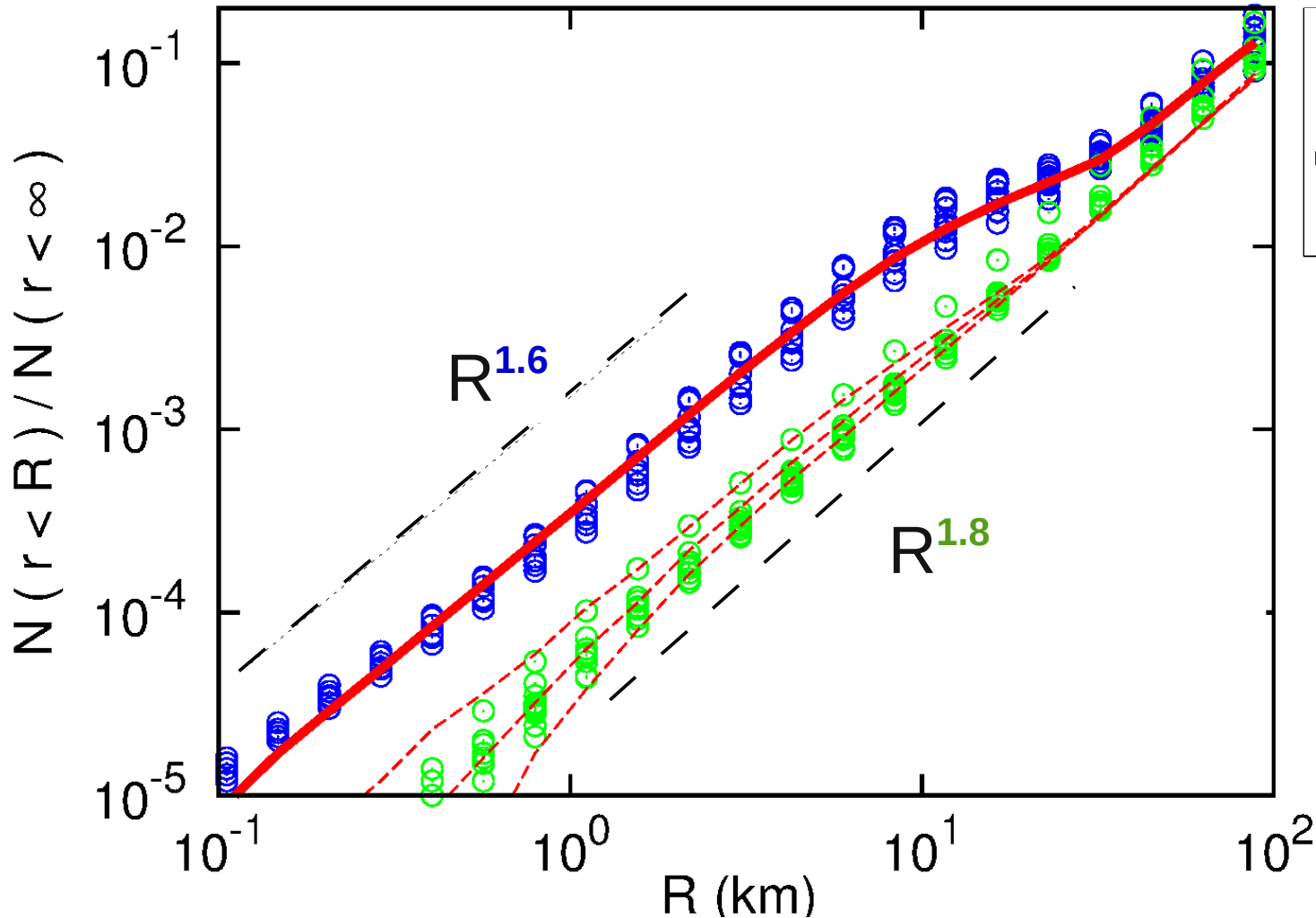
Independent evaluation by means of the ETAS model:  
Is  $d=1.8$  a reasonable value for background activity?



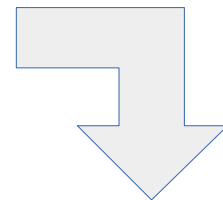
Method based on  
*Beauval et al., BSSA 2006*



Scaling of the correlation integral:  
scaling exponent = fractal dimension



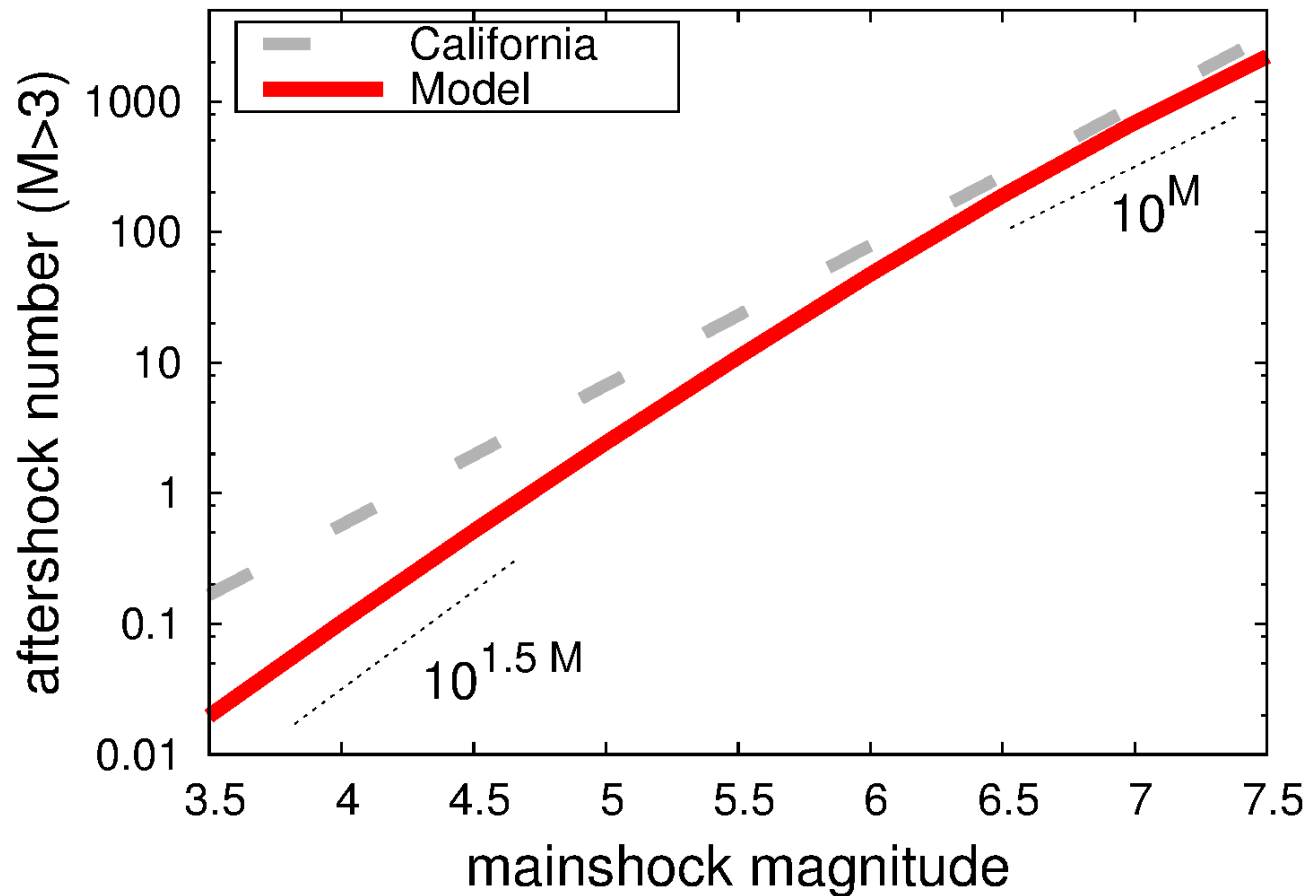
ETAS: all ○  
 declustered ○  
 California: all —  
 declustered - - -



**d=1.8 in  
agreement  
with California  
seismicity**

# Productivity of aftershocks

$$N_a = \int_0^{\infty} \overline{\Delta \text{CFS}(r) H(\Delta \text{CFS})} A(r) W / \overline{M_0} dr$$

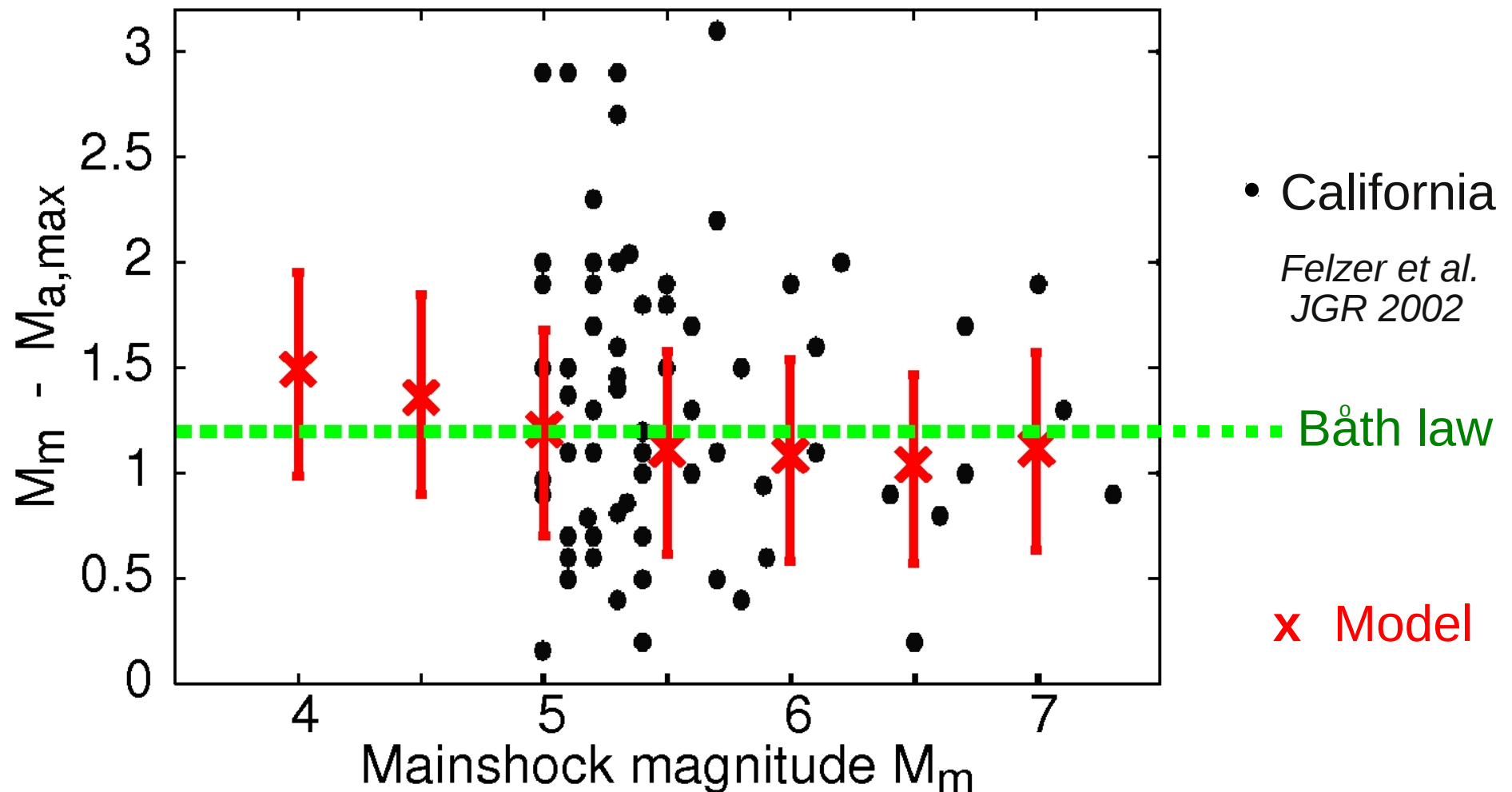


$W = 10 \text{ km}$   
 $b = 1.0$   
 $M_{\min} = -2.0$   
 $M_{\max} = 7.5$

California: *Helmstetter et al., JGR 2005*

## Largest aftershock magnitude as another way to compare with observations:

Empirical observation:  $\langle M_{a,max} \rangle = M_m - 1.2$  (Bath, 1965)



... thus static stress triggering can explain first-order aftershock characteristics regarding total numbers and spatial distribution.

→ static stress changes seem to be the major driving force for aftershock nucleation!

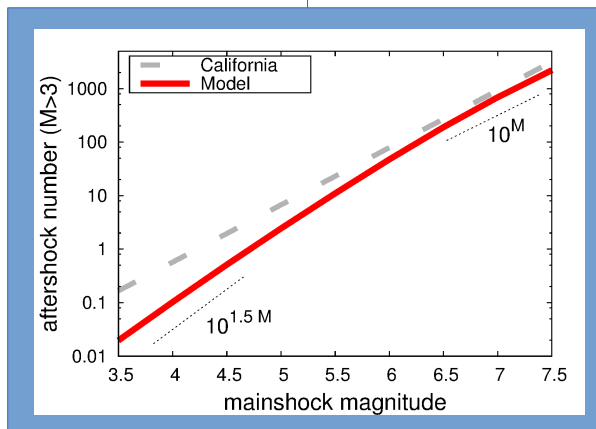
**However**, the model misses so far the **timing of aftershocks** and the contribution of aftershock induced **secondary stress changes**

# One possibility: A statistical model extension **modified ETAS-model:**

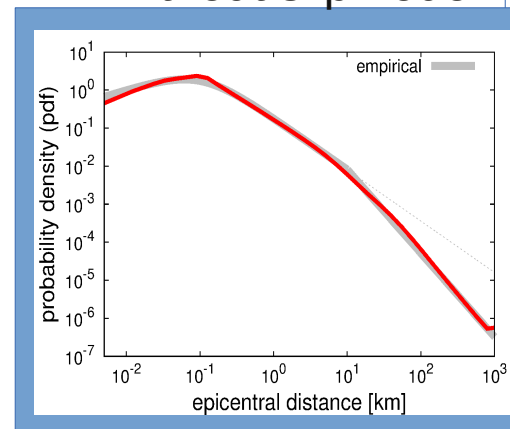
... where productivity scaling and spatial kernel are replaced

$$R(t, \vec{x}) = \mu + \sum_{i:t_i < t} K_0 e^{\alpha M_i} \cdot \frac{1}{(c + t - t_i)^p} \cdot \frac{C_{norm}}{(r^2 + d^2)^{\gamma/2}}$$

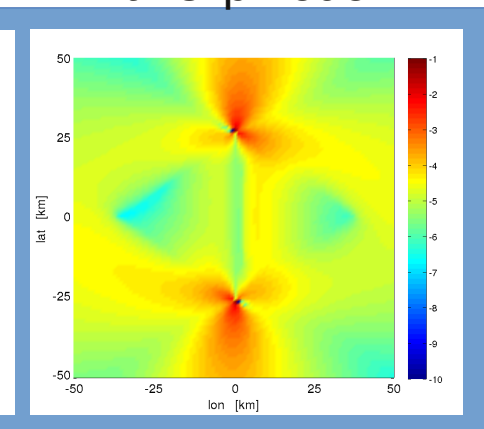
Ogata, 1988



without slipmodel



with slipmodel

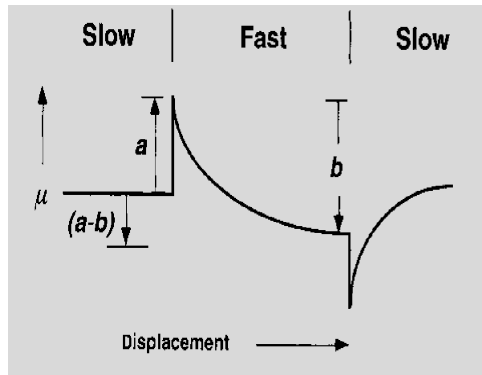


*Bach & Hainzl, JGR 2012; Zakharova et al., JGR 2013*  
... currently tested in CSEP New Zealand

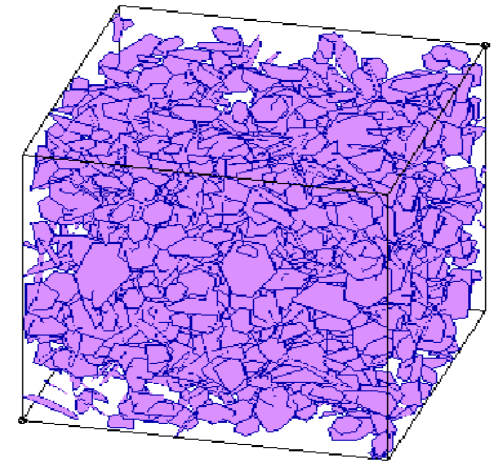
Another possibility:

## Rate- and state-dependent frictional nucleation model

*Dieterich, JGR 1994*



from lab-derived friction law  
 →  
 to statistical response of  
 fault populations



$$R = r / \dot{\tau} \gamma$$

&

$$d\gamma = (dt - \gamma dS) / A \sigma_n$$

stress step:  
 $\Delta S$

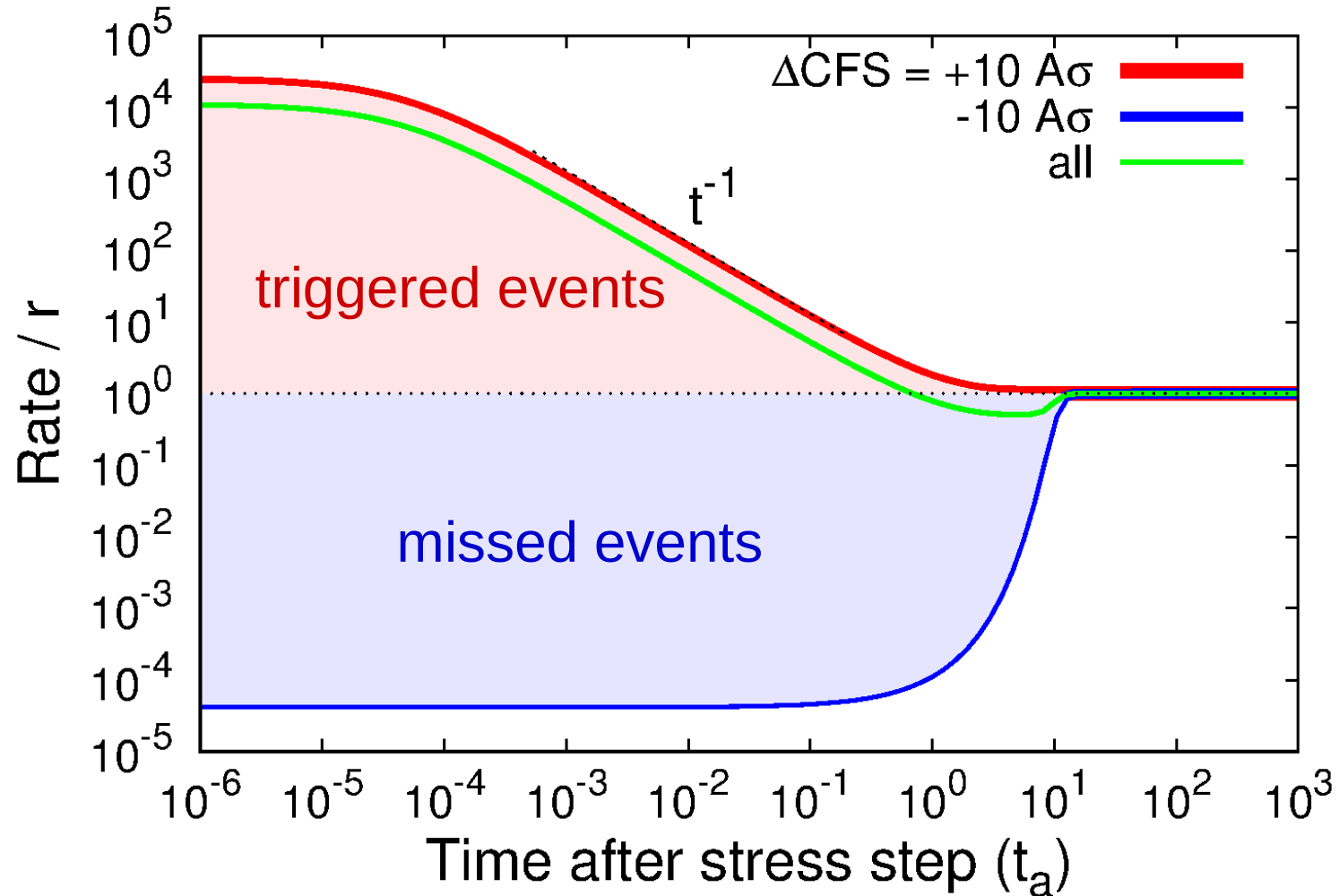
$$R(t, \Delta S) = \frac{r}{1 + \left( e^{-\frac{\Delta S}{A \sigma_n}} - 1 \right) e^{-\frac{t}{t_a}}}$$

$r$  background rate  
 $A$  friction parameter  
 $\dot{\tau}$  tectonic stressing rate

→ relaxation time:

$$t_a = \frac{A \sigma_n}{\dot{\tau}}$$

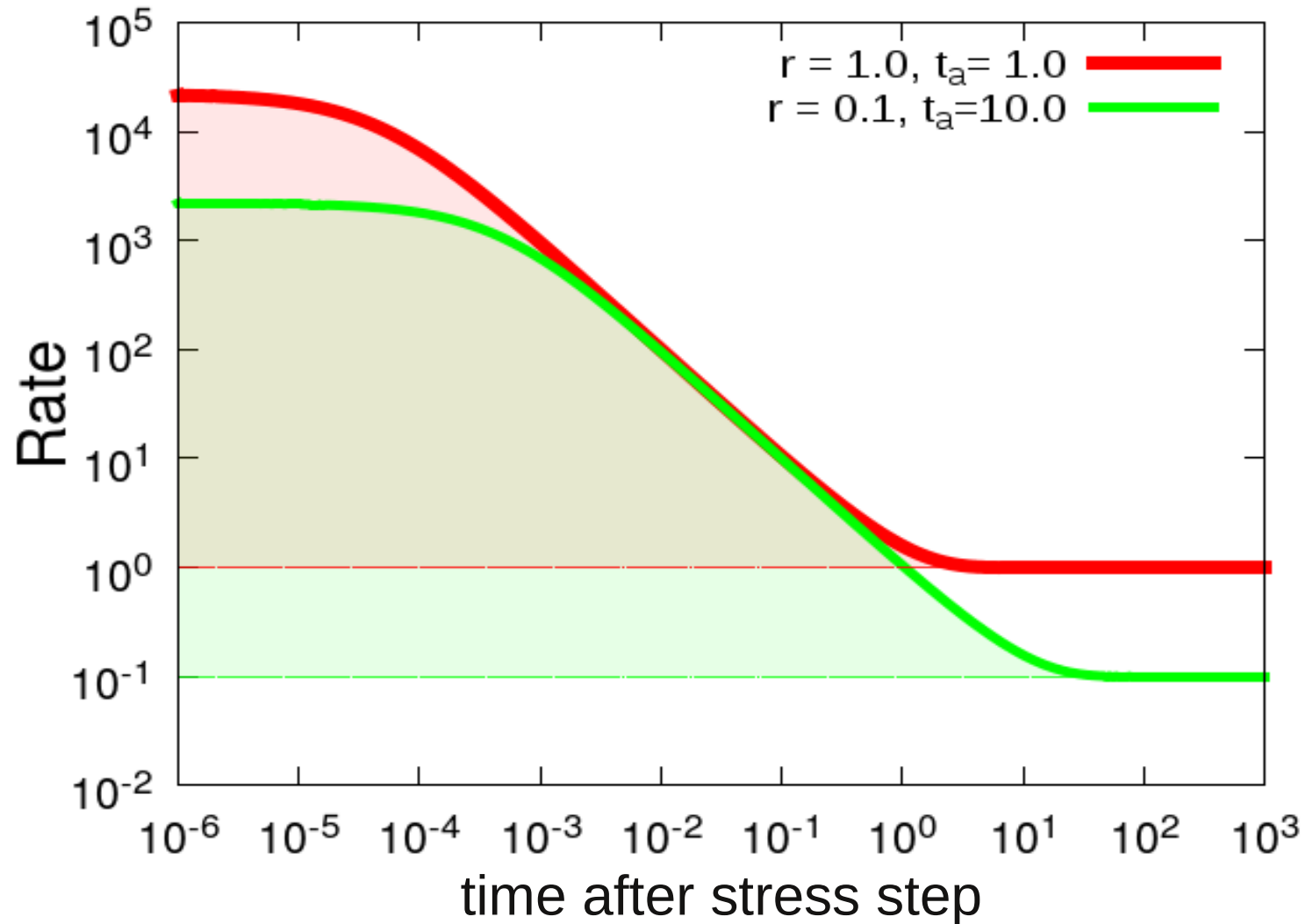
Response to a stress step of  $\pm\Delta\text{CFS}$ :



# triggered ( $+\Delta\text{CFS}$ ) = # missed ( $-\Delta\text{CFS}$ )



In the case of different stressing rates:



$$N_{\text{tot}} = (r/\dot{\tau}) \Delta\text{CFS} \quad (\text{same as in the clock advance model!})$$

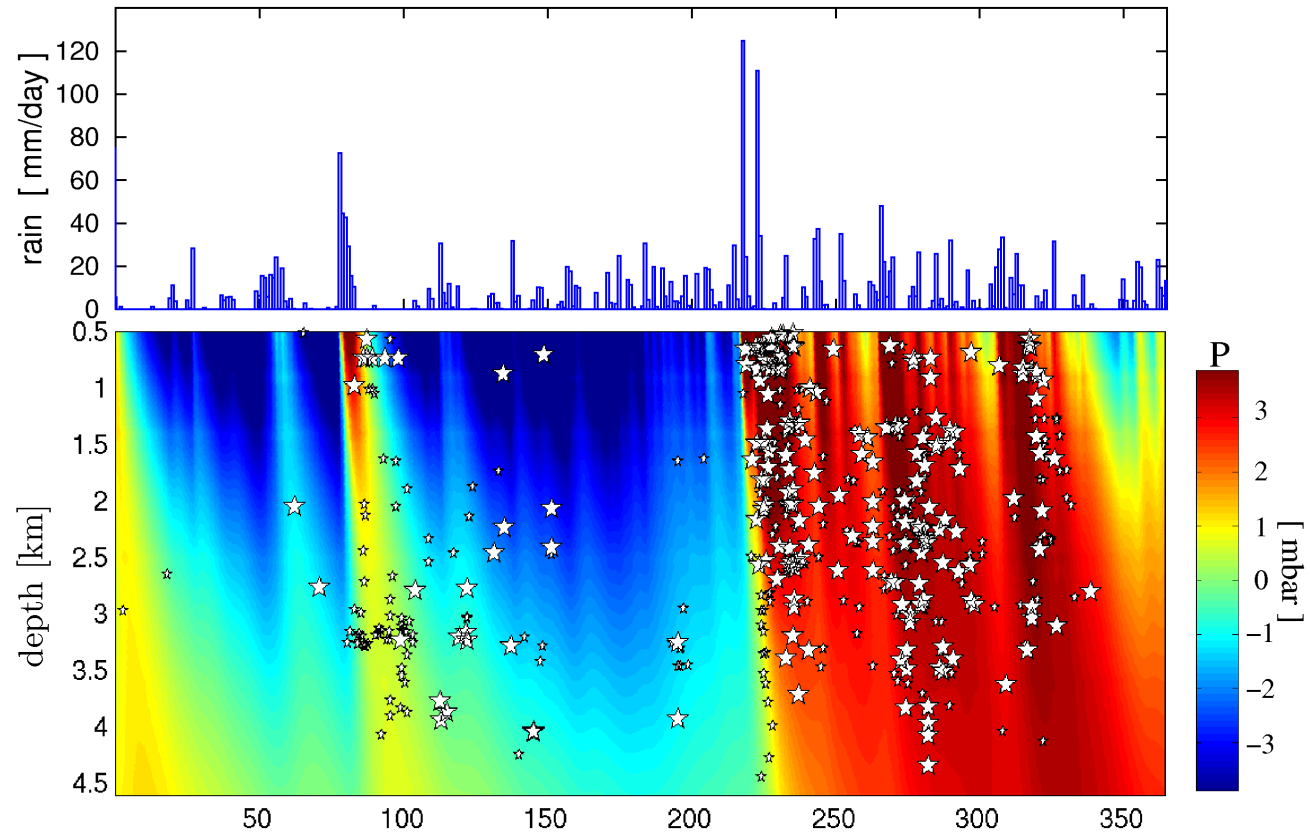
# triggered = # triggered if  $r/\dot{\tau} = \text{constant}$

**Advantage:** Earthquake rate can be determined for complex stress histories resulting from aseismic & coseismic processes

Example:

rainfall

pore pressure  
diffusion



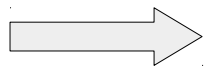
Earthquake activity  
at Mt. Hochstaufen  
SE Germany

*Hainzl et al. GRL 2006*  
*Hainzl et al. JGR 2013*

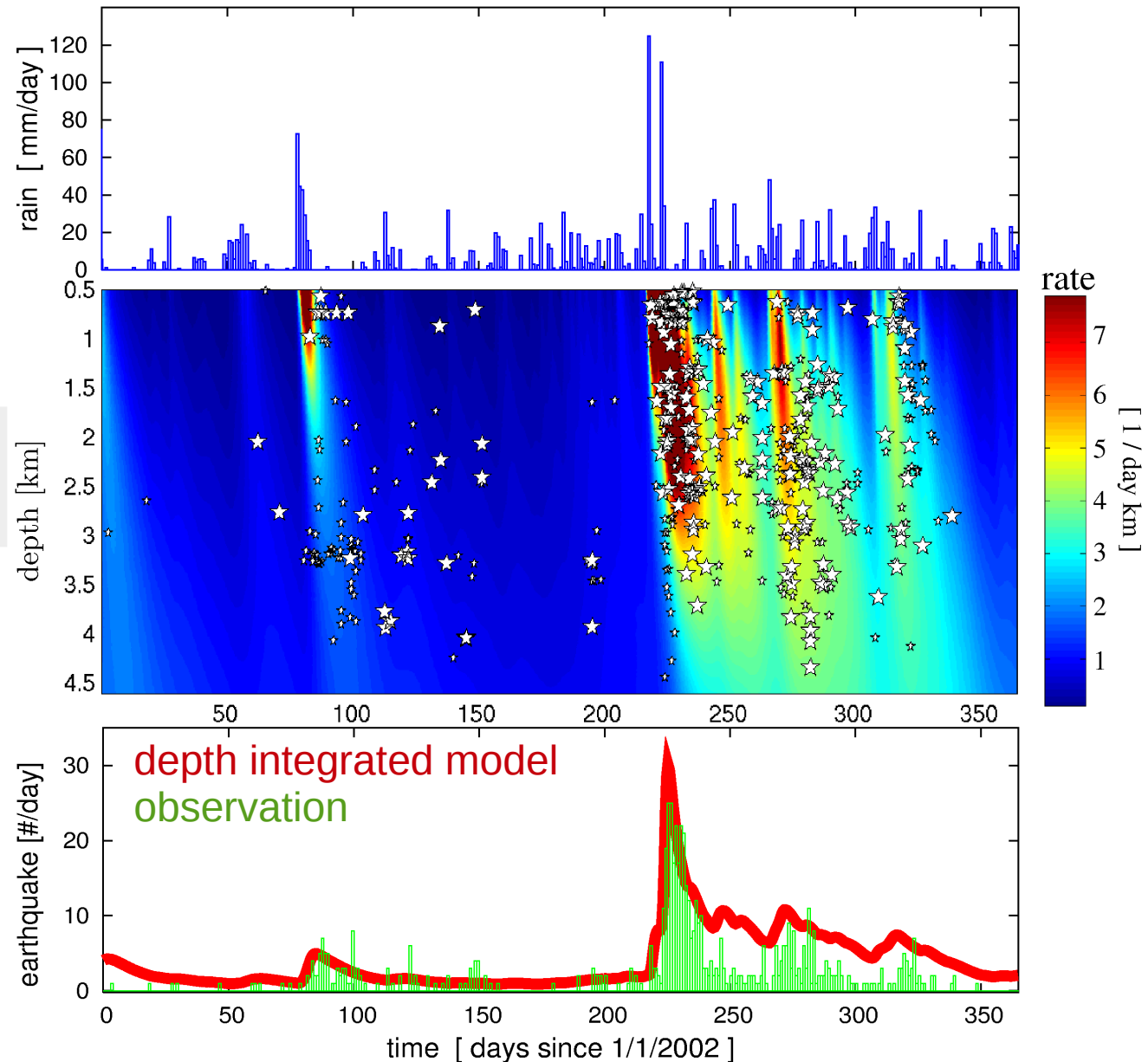
**Advantage:** Earthquake rate can be determined for complex stress histories resulting from aseismic & coseismic processes

Example:

rainfall



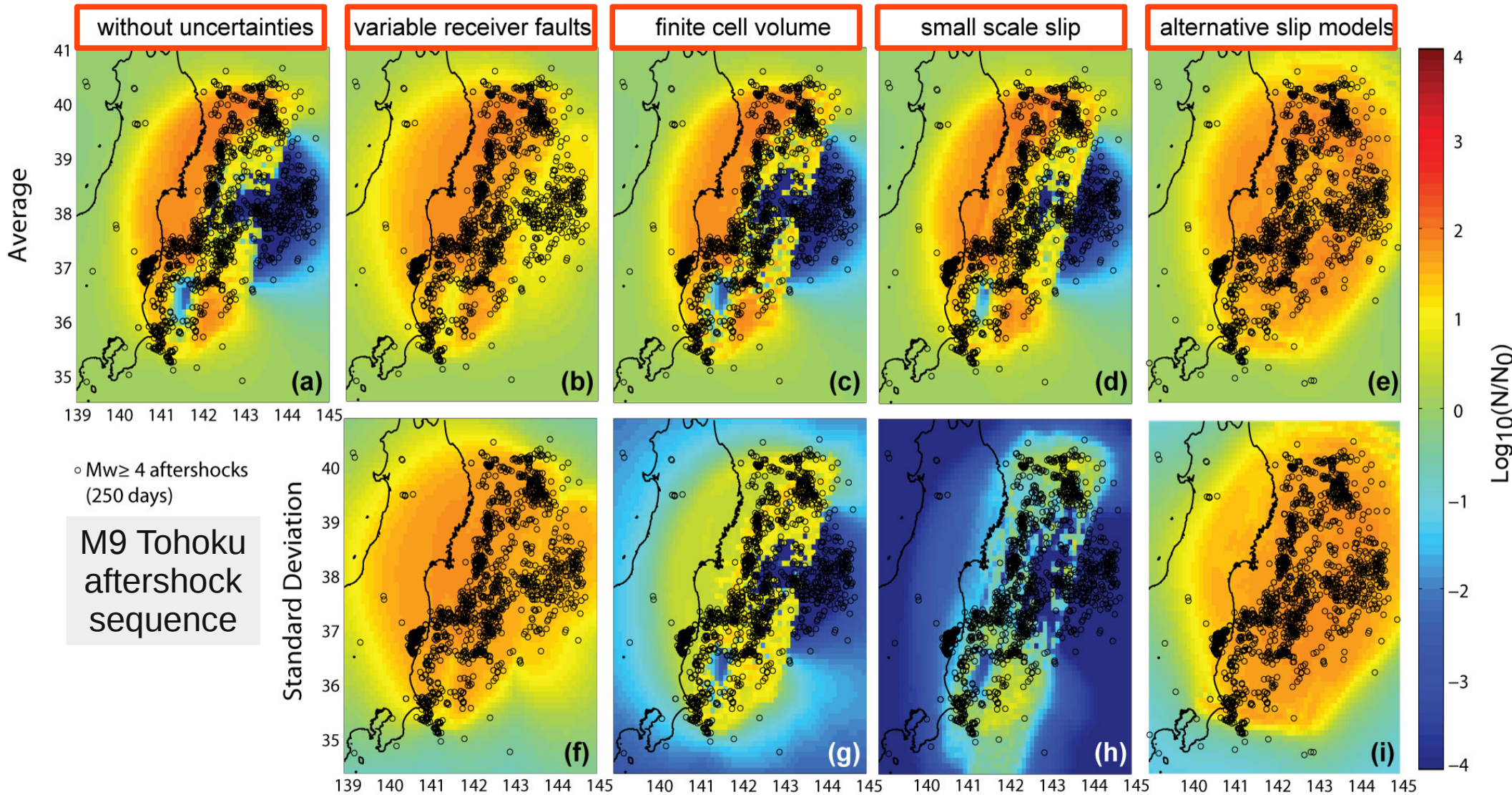
earthquake rate



Earthquake activity  
at Mt. Hochstaufen  
SE Germany

*Hainzl et al. GRL 2006*  
*Hainzl et al. JGR 2013*

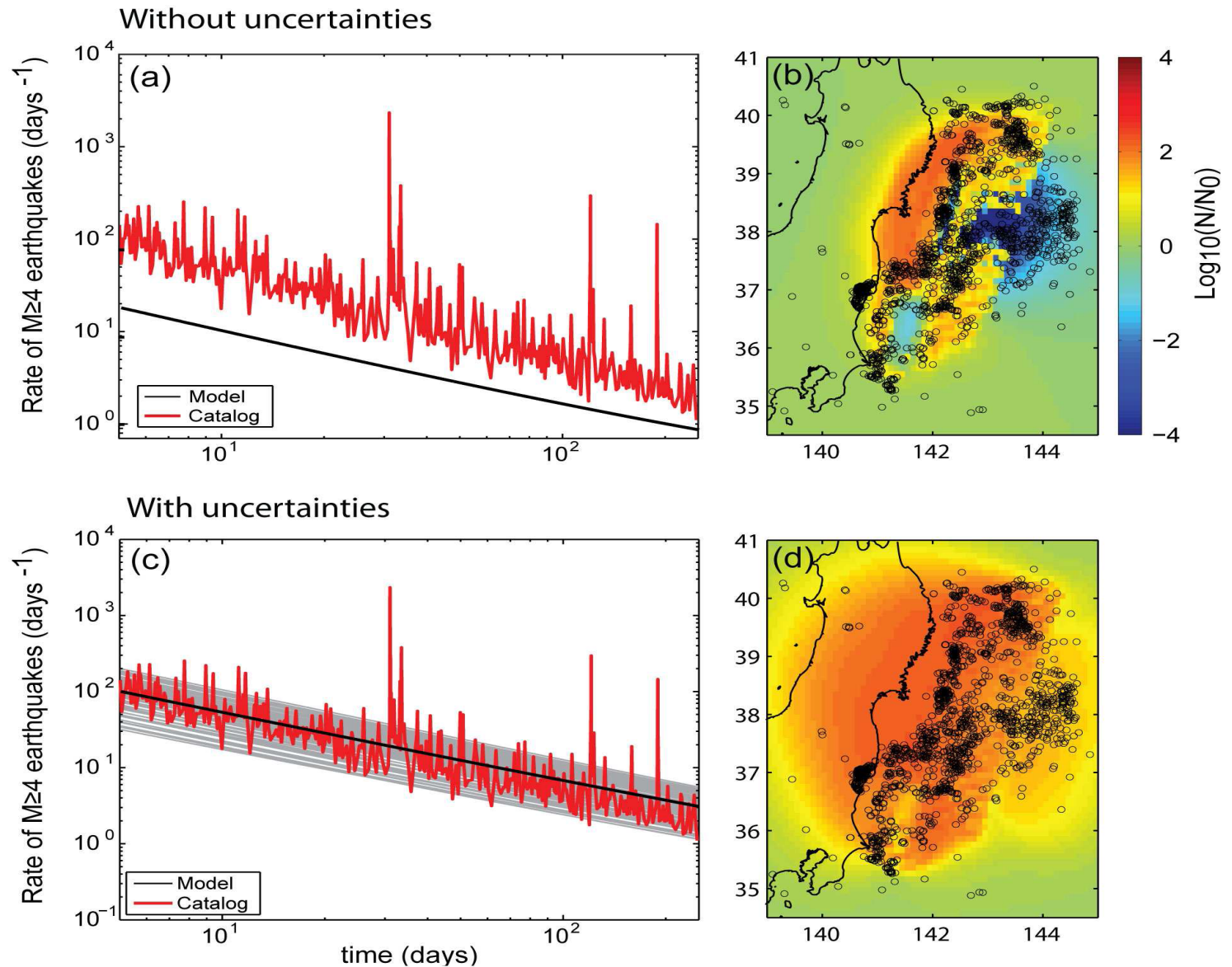
# Challenge: Large uncertainties of $\Delta$ CFS-calculations



Model integration by Monte-Carlo sampling ... *Cattania et al. JGR 2014*



Model performance improves significantly if uncertainties are systematically taken into account:



M9 Tohoku  
aftershock  
sequence  
*Cattania et al.*  
*JGR 2014*

## Summary

Static stress triggering can explain first-order characteristics:

- spatial distribution
- productivity
- Omori aftershock decay

However:

Uncertainties of the actual stress state (use of Poisson model for the initial state) and the  $\Delta$ CFS-calculations are large and limit the forecast ability.

Proper accounting for uncertainties is necessary for hypothesis testing and improved forecasts.

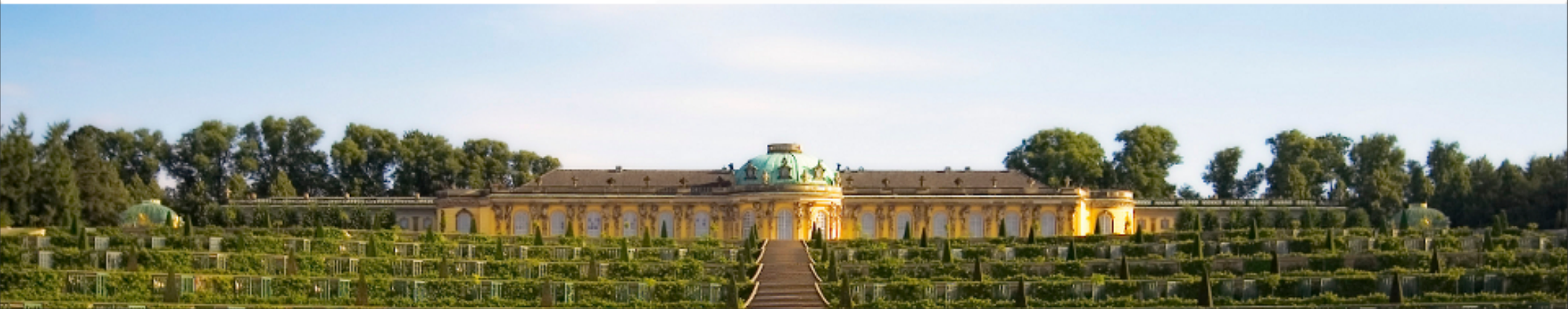
# Thank you!



StatSei9

Potsdam, June 14-19, 2015

<https://statsei9.quake.gfz-potsdam.de>



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## 9th International Workshop on Statistical Seismology

The International Statistical Seismology (StatSei) workshop is a well-established biannual meeting focusing on recent developments in statistical seismology. The meetings covered a series of topics, including statistical descriptions of earthquake occurrences, earthquake physics, earthquake forecasting and forecast evaluations, earthquake triggering and many more. Starting in 2001, it was held in China, Japan, New Zealand, Mexico, United States, Italy and Greece. The 9th StatSei workshop will be hosted by the GFZ Potsdam (German Research Center for Geosciences) and the University of Potsdam.

We invite statisticians and seismologists as well as geologists working on theoretical and applied questions of seismology to join the workshop in the beautiful historic environment of Potsdam.

We are looking forward to welcome and host you in Potsdam!

The organizing committee

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