

# **Link between seismicity and deformation: Application to the postseismic phase**

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support on this work

# Outline of the talk

- 1) The Coulomb failure stress  $\Delta CFF$
- 2) Coseismic Coulomb stress changes, aftershocks and optimally oriented planes
- 3) Is the Coulomb failure function compatible with more realistic friction laws, i.e., rate and state friction?
- 4) Dieterich's model of seismicity
- 5) Characteristics of afterslip
- 6) Afterslip model of seismicity
- 7) Case of the Landers earthquake
- 8) Case of the Tohoku earthquake
- 9) Conclusions



# The Coulomb Failure Model

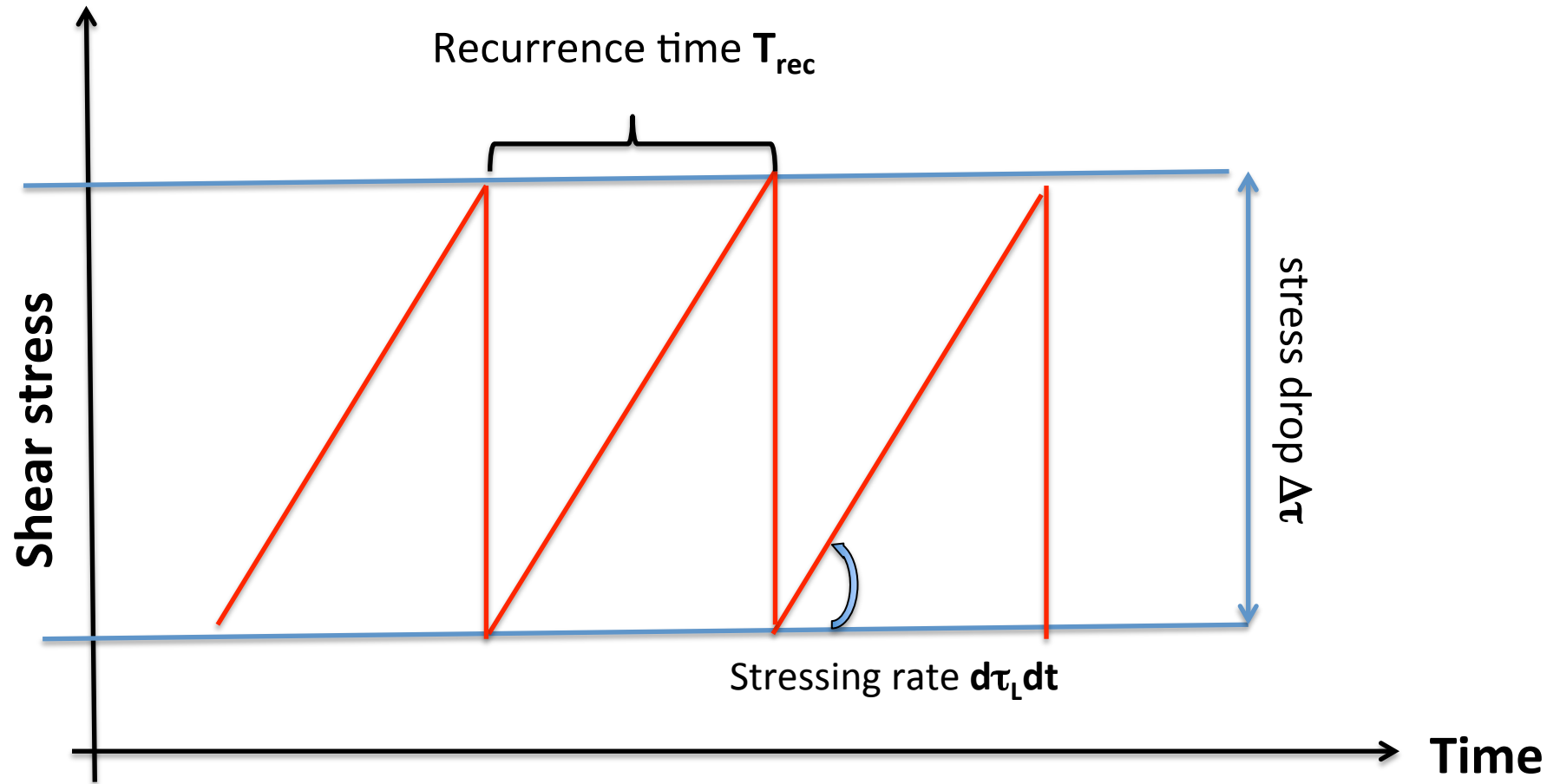
$$\Delta CFF = \Delta\tau - \mu\Delta\sigma$$

$\Delta\tau$ : shear stress change

$\Delta\sigma$ : normal stress change (<0 for unclamping)

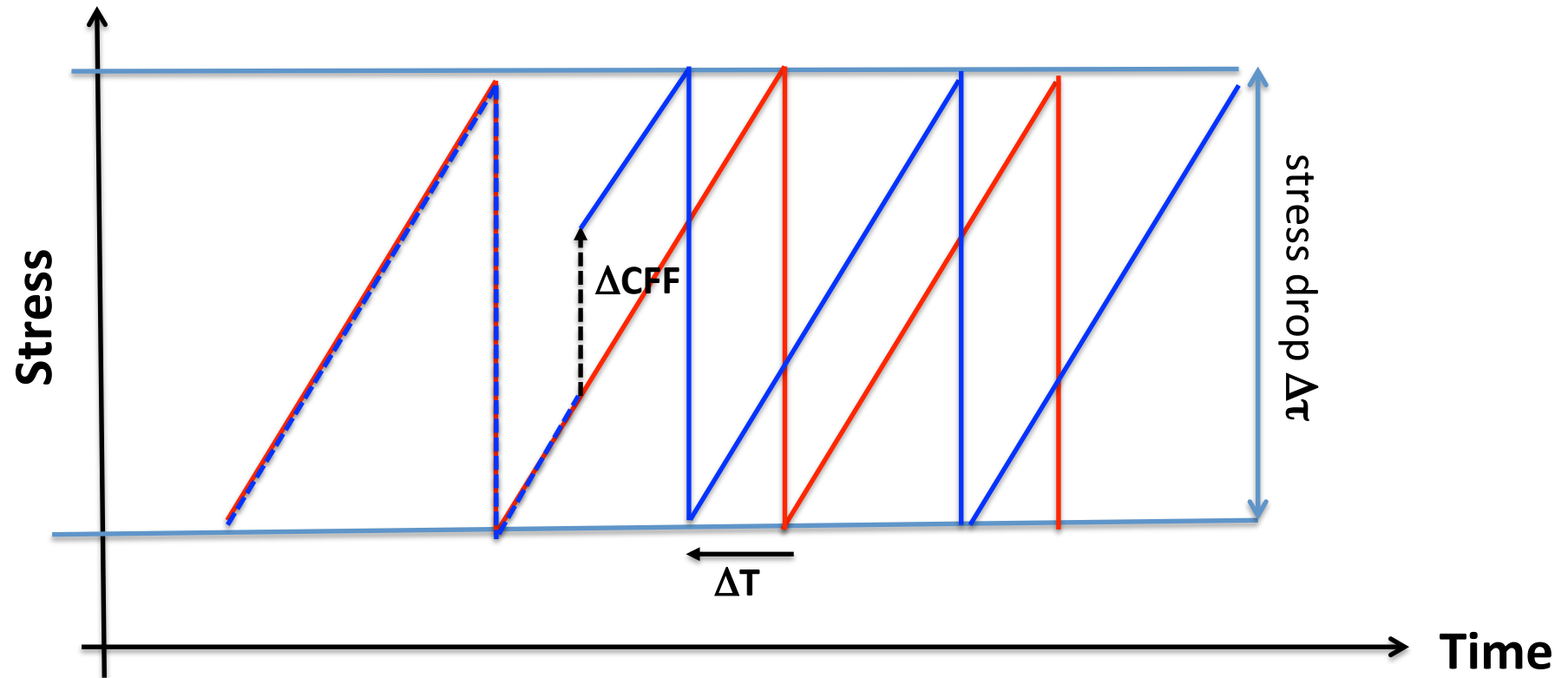
$\mu$ : (constant) friction coefficient

# The Coulomb Failure Model: Clock Change



$$T_{rec} = \Delta\tau / (d\tau_L/dt)$$

# The Coulomb Failure Model: Clock Change (I)



Clock change:  $\Delta T = \Delta CFF / (d\tau_L / dt)$

Independent of the time of the applied perturbation

Instantaneous triggering if  $\Delta CFF > \Delta\tau$

# The Coulomb Failure Model: Clock Change (II)

Clock change:  $\Delta T = \Delta CFF / (d\tau_L / dt)$

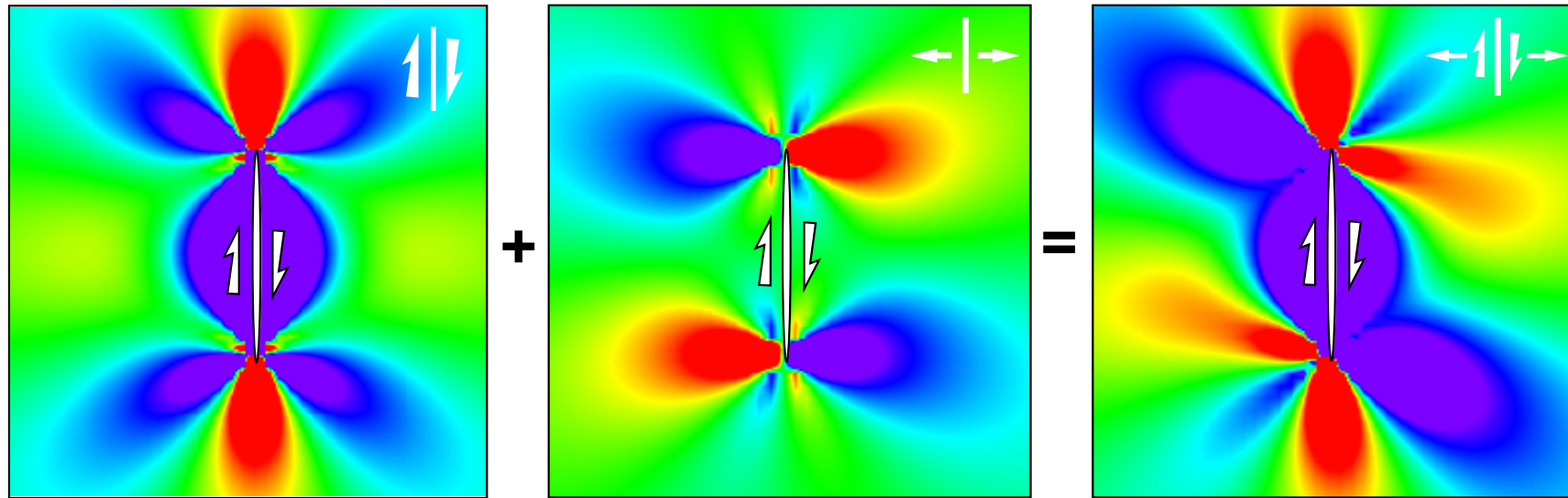
$\Delta CFF > 0$ : Clock advance  $\rightarrow$  Triggering expected

$\Delta CFF < 0$ : Clock delay  $\rightarrow$  Triggering not expected

# The Coulomb Failure Model and Coseismic Stress Changes

A. Coulomb stress change for *right-lateral faults parallel to master fault*

Stress ■ Rise ■ Drop



right-lateral shear  
stress change

$\Delta\tau$

+

effective friction  $\times$   
normal stress change

+

$\mu \Delta\sigma$

=

right-lateral Coulomb  
stress change

=

$\Delta\text{CFF}$

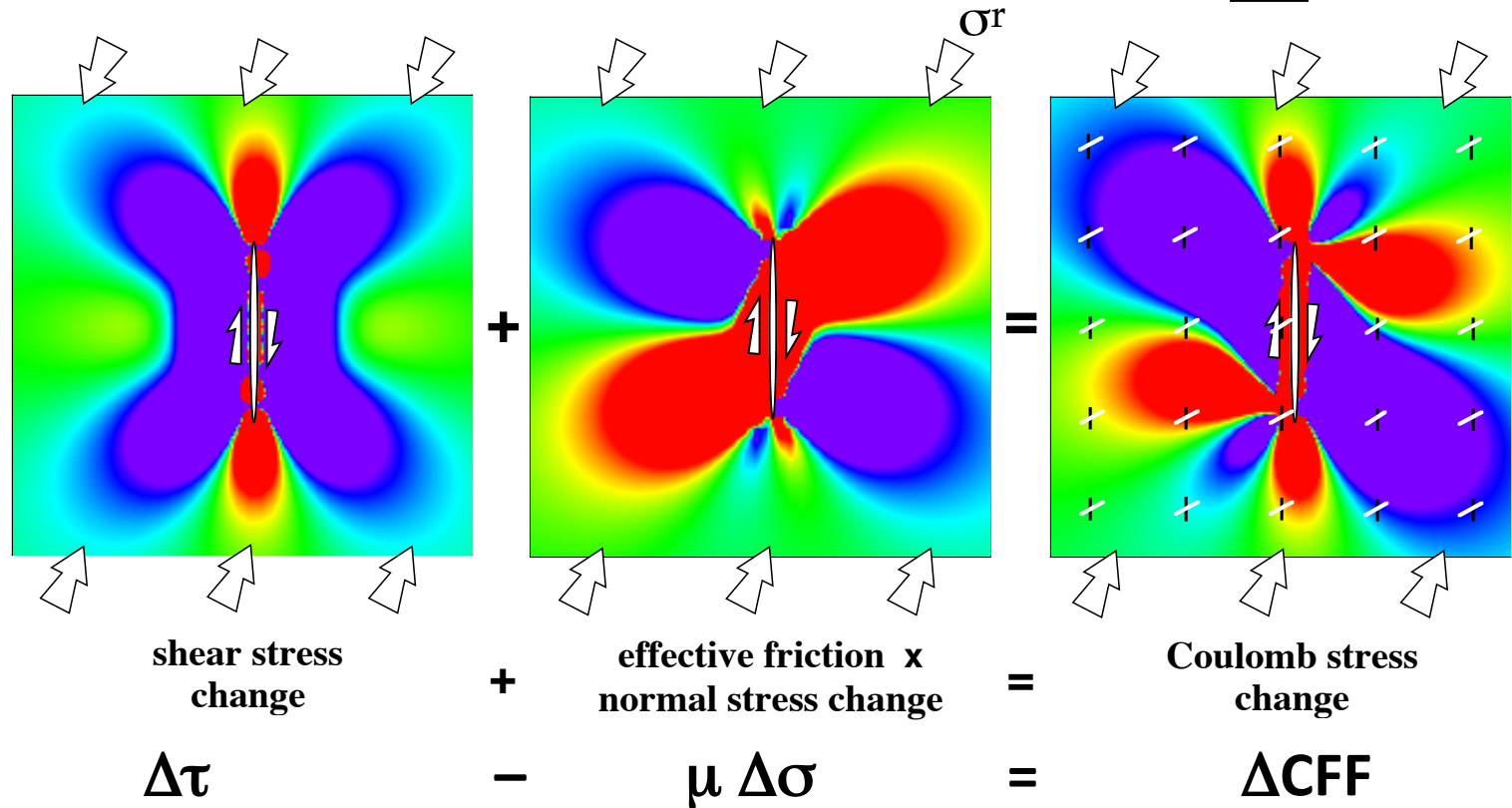
King et al., 1994

Coulomb stress computed assuming that each receiver fault (i.e., a grid point) is oriented as the main fault

# The Coulomb Failure Model and Coseismic Stress Changes: Optimally Oriented Planes

B. Coulomb stress change for faults optimally oriented for failure  
 N27°E regional compression ( $\sigma^r$ ) of 100 bars;  $\mu' = 0.75$

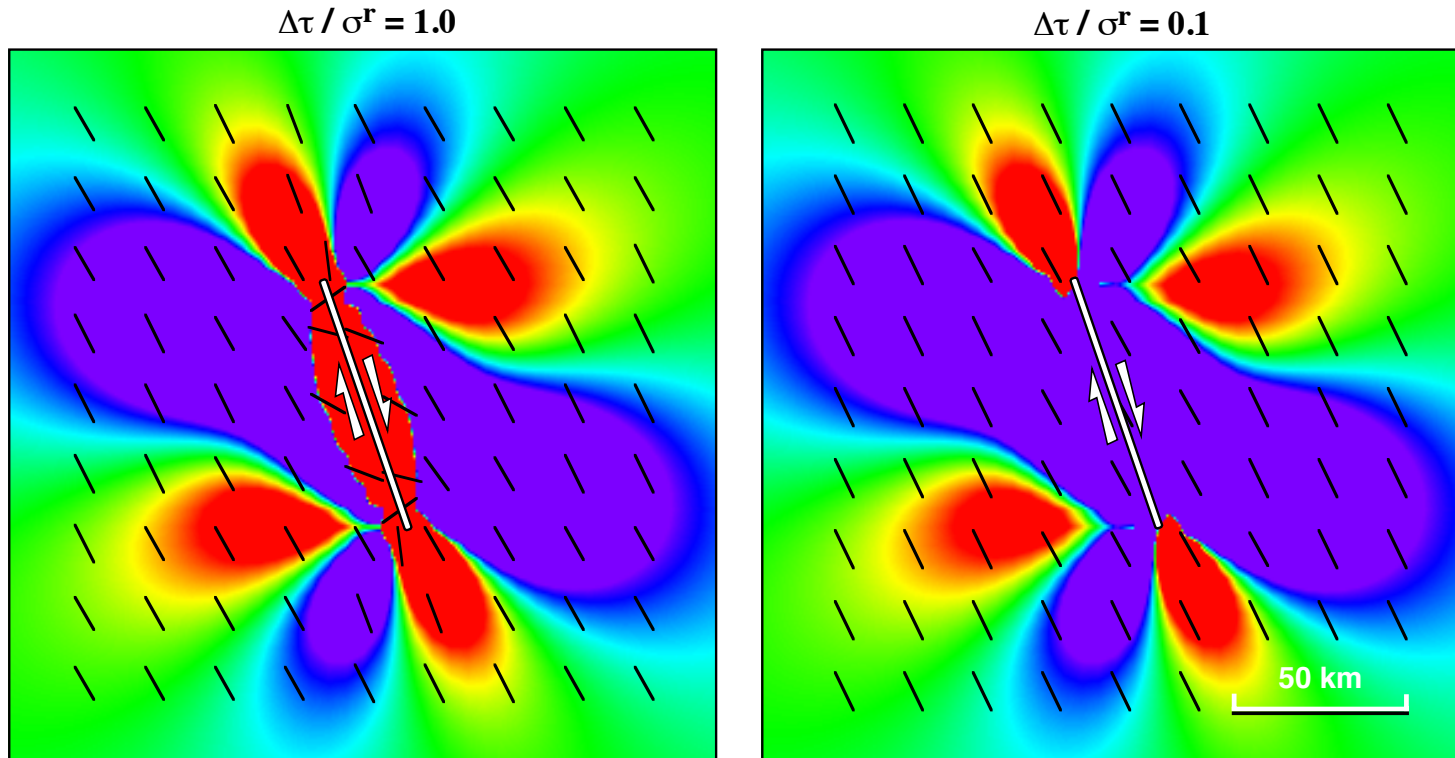
Optimum Slip Planes  
 left-lateral  
 right-lateral



King et al., 1994

Coulomb stress computed assuming that each receiver fault (i.e., a grid point) is oriented optimally to get the maximum  $\Delta\text{CFF}$

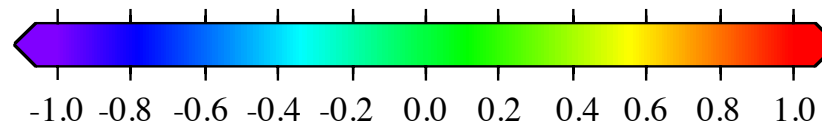
# The Coulomb Failure Model and Coseismic Stress Changes: Optimally Oriented Planes



King et al., 1994

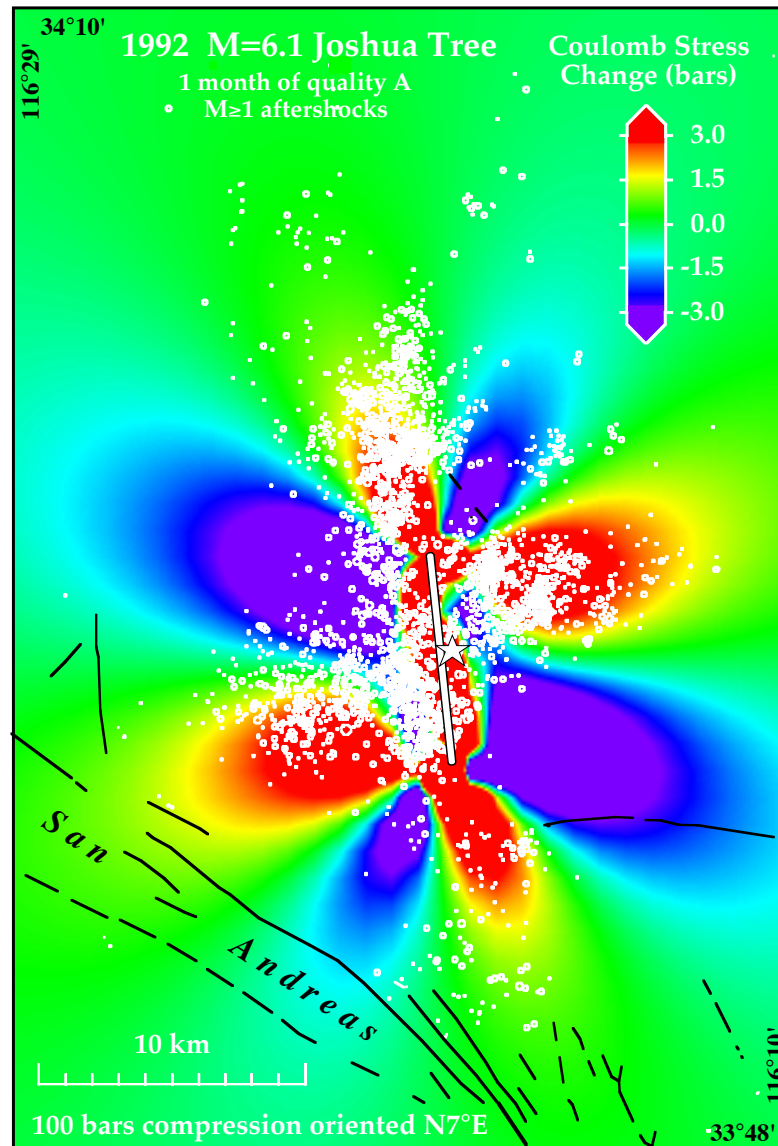
Change in Coulomb Stress (bars) on  
optimal right-lateral faults (black)

$\sigma^r$  oriented N7°E,  $\mu = 0.4$



- In the far field, the OOP are oriented conformably to the tectonic field
- In the near field, they follow the coseismic stress field if  $\Delta\tau \gg \tau^r$ , where  $\sigma^r$  is the regional tectonic stress field

# Application to the 1992, Mw7.3, Landers earthquake

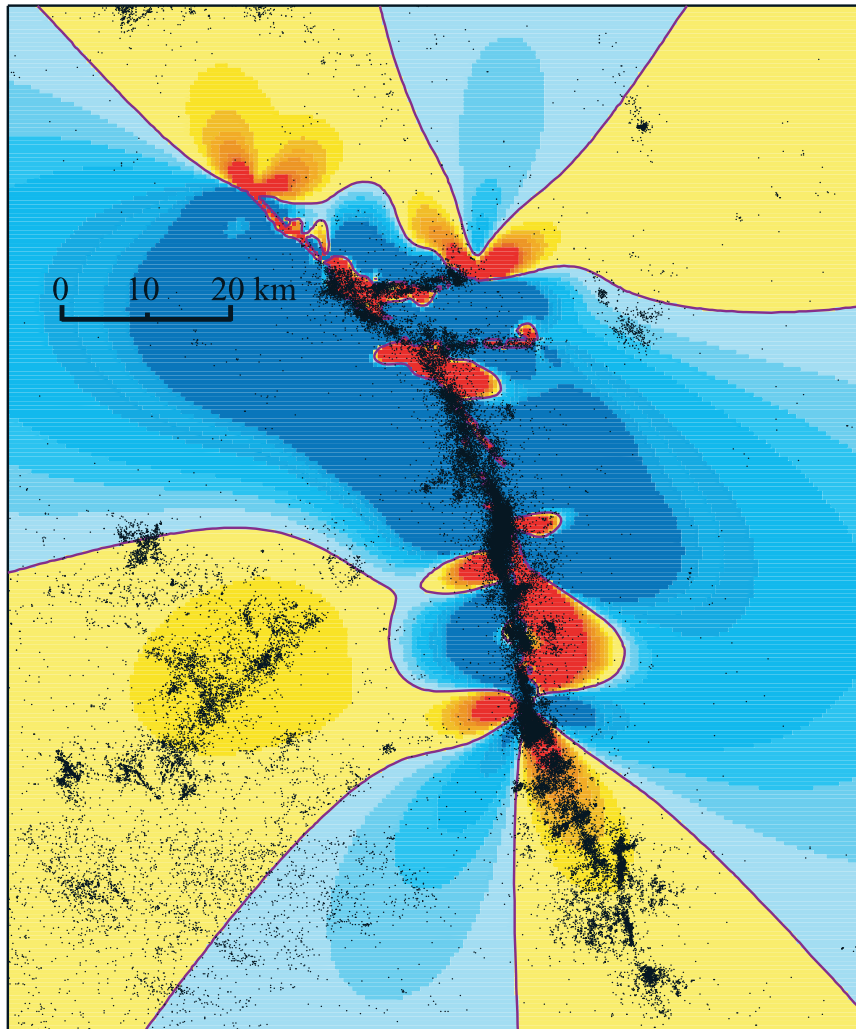


King et al., 1994

Good correlation between the distribution of aftershocks and the coseismic Coulomb stress change assuming the OOP

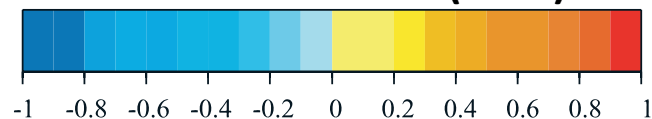


# Application to the 1992, $M_w$ 7.3, Landers earthquake



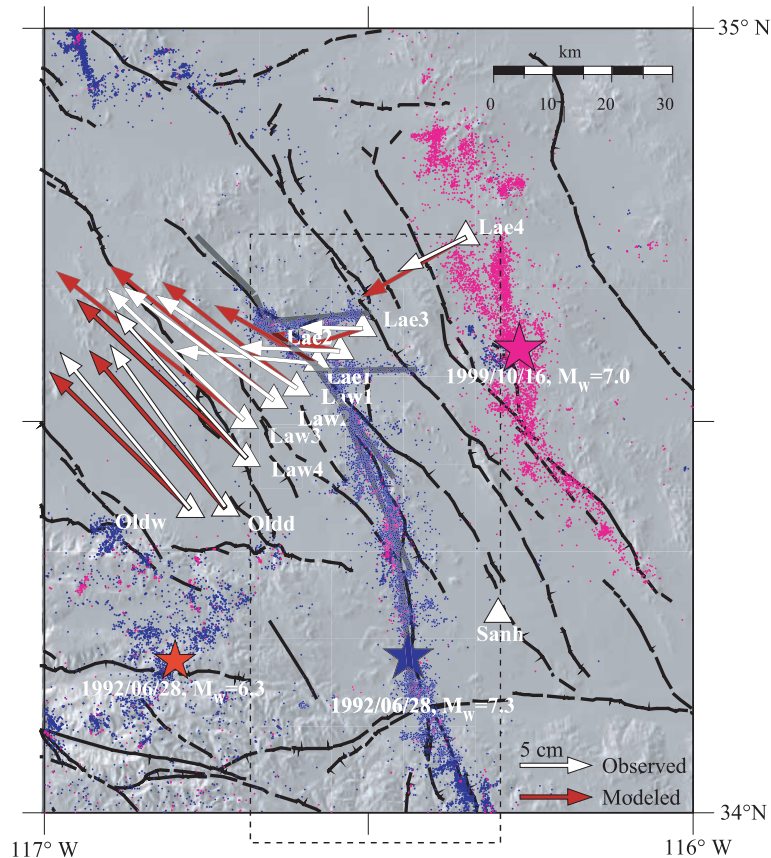
*Perfettini and Avouac, 2007.*

**ΔCFF Coseismic (MPa)**



- Weak correlation between the distribution of aftershocks and the coseismic Coulomb stress change assuming that all receiver faults are oriented parallel to the main fault
- The coseismic stress field near the fault highly depends on the details of the coseismic slip model

# Optimally Oriented Planes

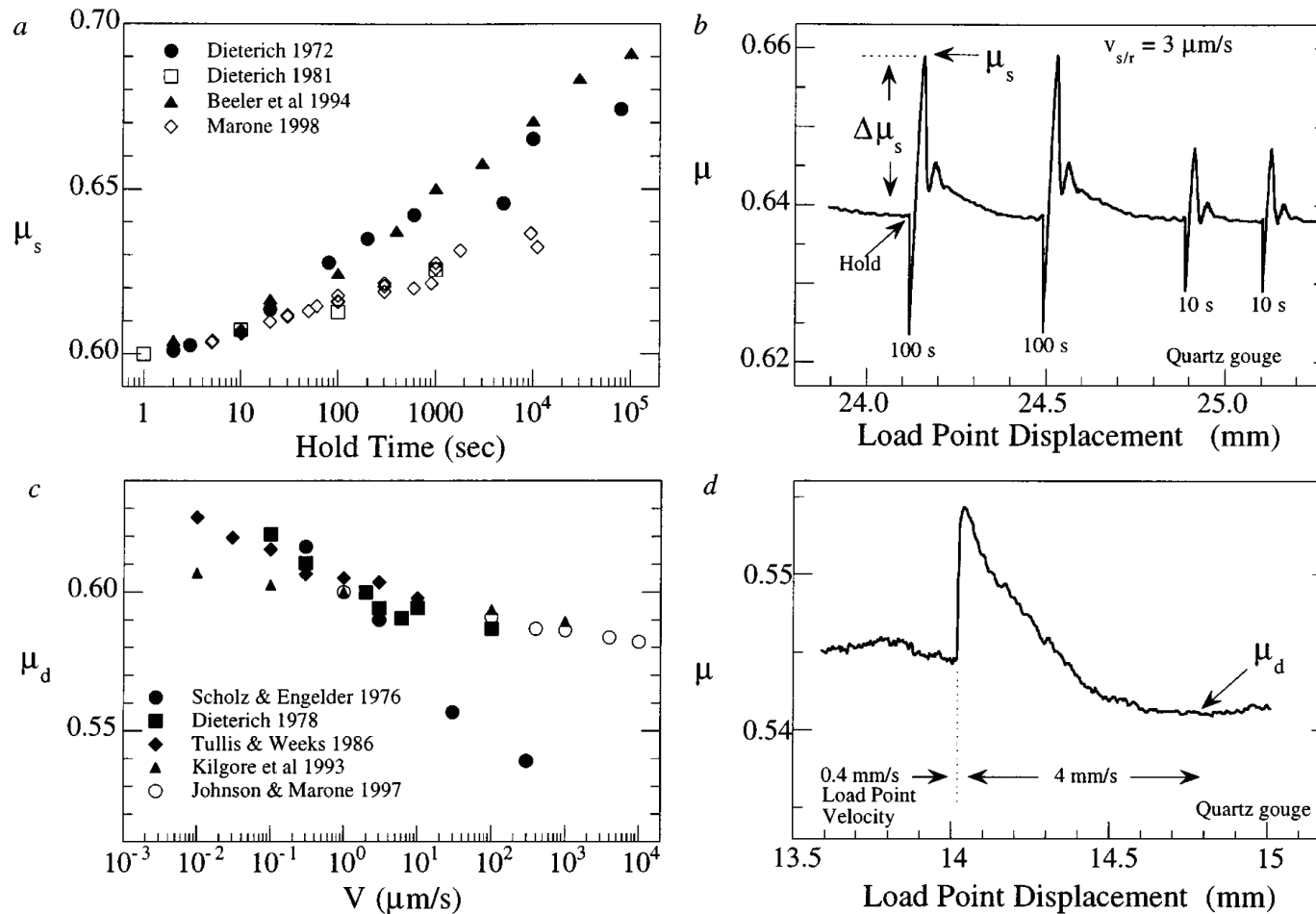


*Perfettini and Avouac, 2007.*

- The tectonic map of the Landers earthquake show that most of the structures are oriented as the main fault
- The focal mechanisms of the aftershocks show at most a 15° variations from their pre-earthquake value

- Is it possible that new fault interface can be created knowing that a M<sub>w</sub>4 earthquake corresponds to a radius of about 600m?
- Imagining that new faults are created, they would be created unloaded. Why should they re-rupture so quickly?
- The concept of OOP might be acceptable for small aftershocks (M<sub>w</sub><2) but not for large aftershocks

# Rate and State friction



Marone, 1998.

- $\tau/\sigma = \mu = \mu_* + a \ln(V/V_*) + b \ln(\theta/\theta_*)$ 
  - $d\theta/dt = 1 - V\theta/d_c$  (aging law)
- Steady-state sliding:  $\theta_{ss} = d_c/V \Rightarrow \mu_{ss} = \mu_* + (a-b) \ln(V/V_*)$

$$\mu_{ss} = \mu_* + (a-b) \ln(V/V_*)$$

$a-b > 0$

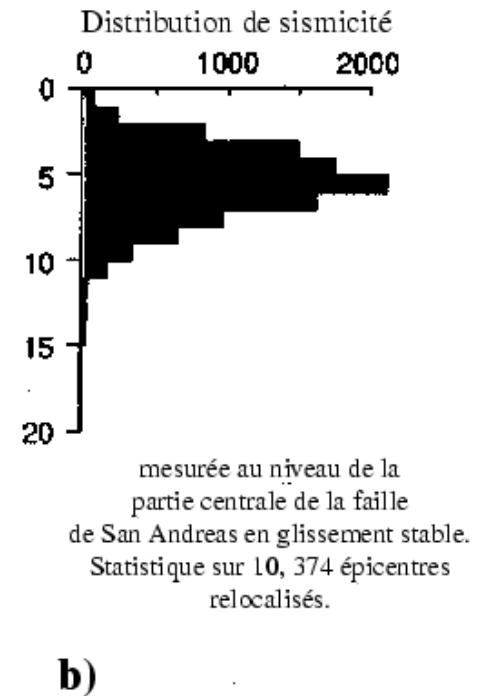
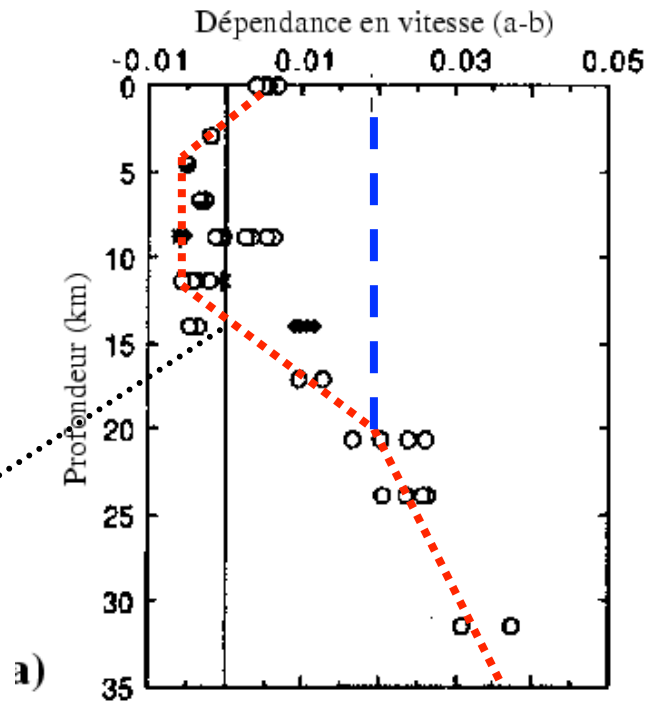
$a-b < 0$

**Stable slip:** creeping faults

**Potentially unstable slip:** unstable faults

## Explain the confinement of seismicity with depth

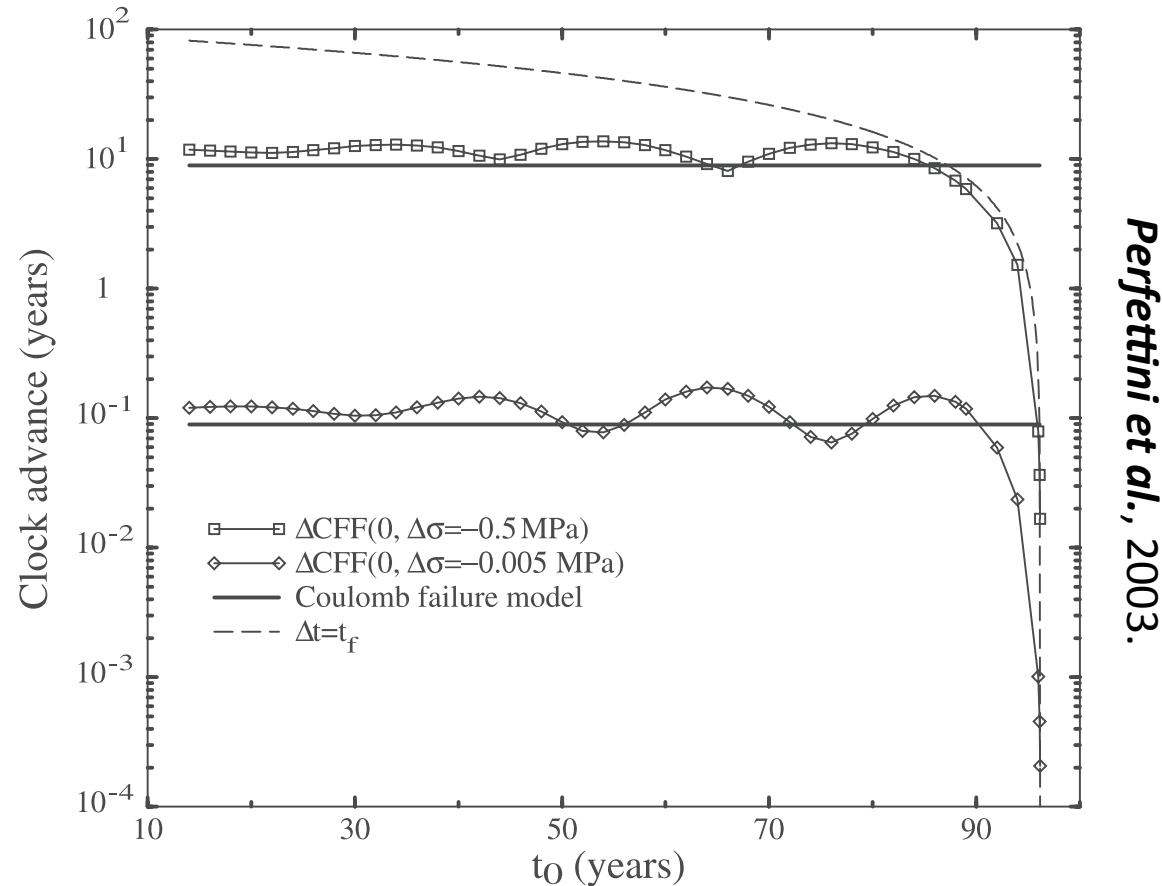
..... a-b  
 --- a



Correspond à  $T \sim 350^\circ\text{C}$

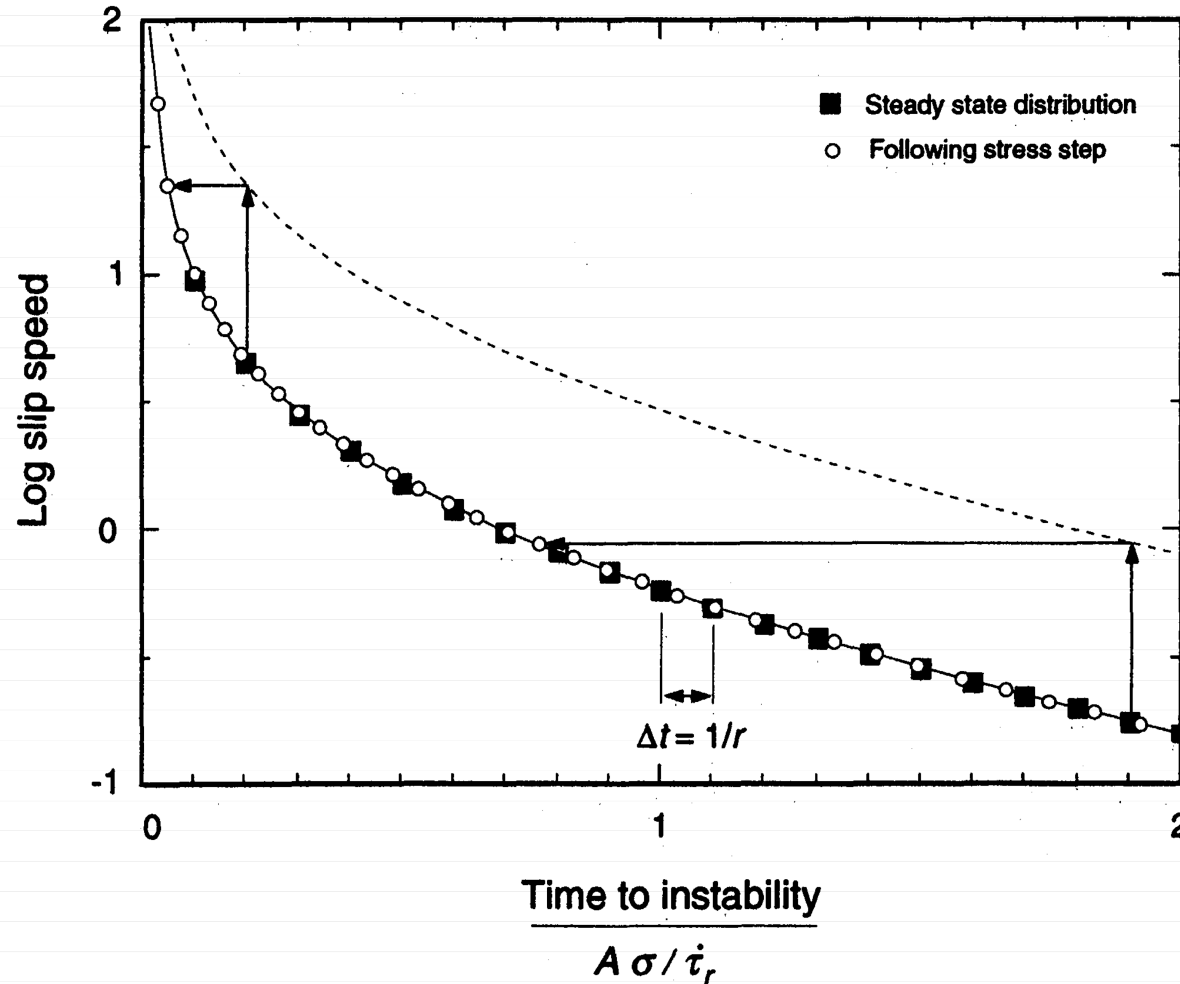
*Blanpied et al., 1991.*

# Clock change and agreement between Rate and State friction and the Coulomb Failure model



- The Coulomb failure model agrees with the prediction of rate and state friction during most of the cycle, except at the end of the earthquake cycle, when nucleation is underway

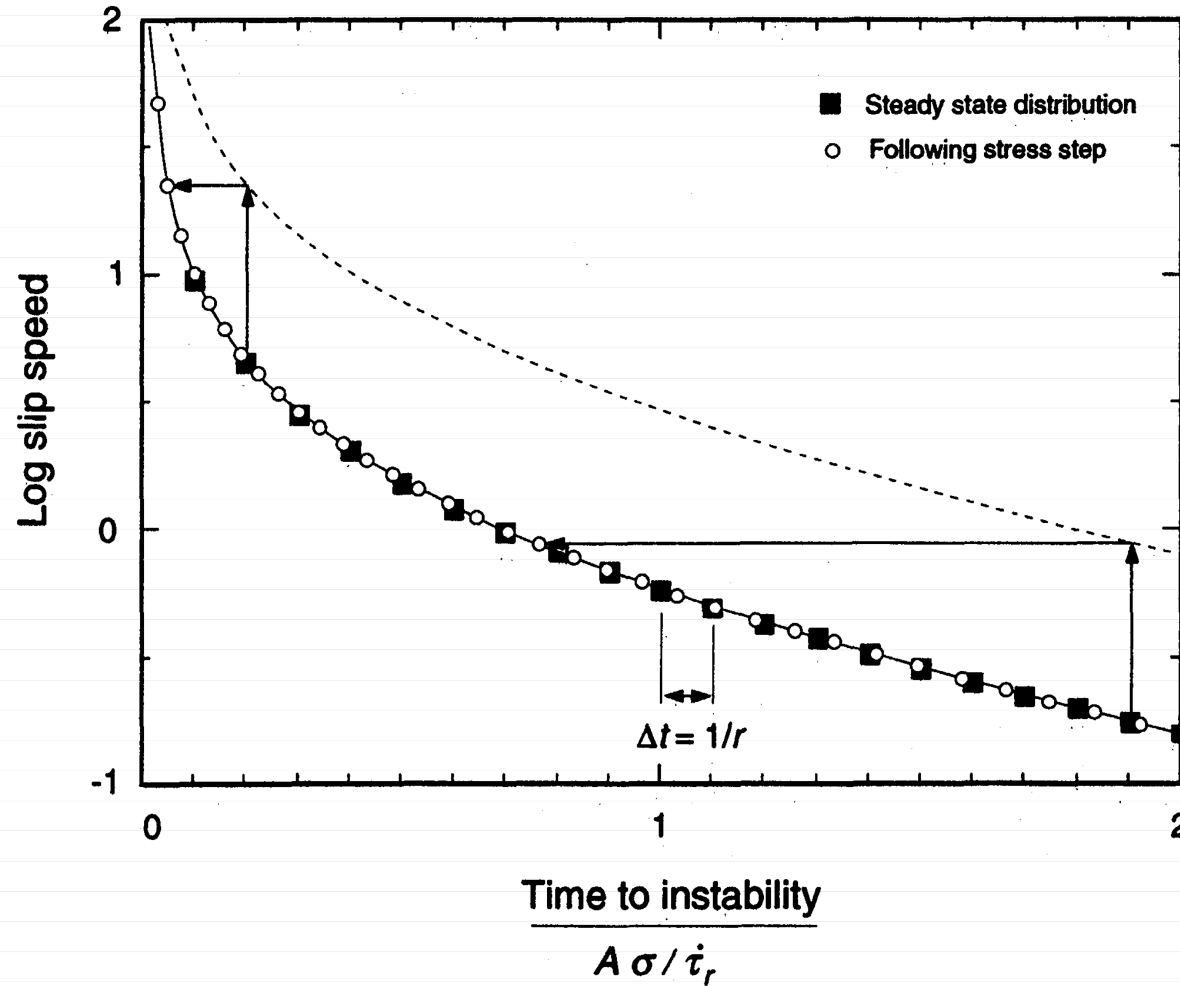
# Dieterich's Model of Seismicity



Dieterich, 1994.

- Distribution of initial velocities built to result in a constant seismicity rate  $R_L$  in the case where  $\Delta CFF=0$
- Change of the nucleation time of a population of faults due a stress step that changes the sliding velocity:  $V^+=V^- \exp(\Delta CFF/a\sigma)$

# Dieterich's Model of Seismicity



Dieterich, 1994.

- The stress change brings each elementary fault closer to failure
- The earlier in the cycle the perturbation is applied, the larger the clock advance
- The hypothesis of a constant seismicity rate is imposed and does not arise naturally

## Dieterich's Model of Seismicity

$$\mathbf{R}(t) = \mathbf{R}_L / [\gamma(t) \, d\tau_L / dt]$$
$$d\gamma / dt = (1 - \gamma \, d\tau / dt) / (a\sigma)$$

**R**: seismicity rate

$d\tau_L / dt$ : long term loading rate

$\mathbf{R}_L$ : long term seismicity rate

**a**: rate and state parameter

$\sigma$ : effective normal stress

- In steady-state:  $\gamma = 1 / d\tau_L / dt$  and  $\mathbf{R} = \mathbf{R}_L$
- Surprisingly, the parameter **b**, responsible for instability in the rate and state framework disappear in Dieterich's model....
- Only the parameter **a**, responsible for the “viscous effect” of rate and state friction shows up...



# Integral Formulation of Dieterich's Model

$$\Delta N(t) = N(t) - N(0) = R_L t_a \ln[1 + (R(0)/R_L)(I(t)/t_a)]$$

with

$$R(0) = R_L / (\gamma(0) d\tau_L dt)$$

$$I(t) = \int_0^t dt' \exp[(\tau(t') - \tau(0))/A]$$

$$t_a = A / d\tau_L dt$$

with  $A = a\sigma$

The most general form for the evolution of the shear stress following a mainshock is

$$\tau(t) = \tau_0 + d\tau_L dt \cdot t + \Delta\tau_{\text{cos}}(t) + \Delta\tau_{\text{post}}(t)$$

## Application to a coseismic step

$$\tau(t) = \tau_0 + d\tau_L dt \cdot t + \Delta\tau_{\text{cos}} H(t)$$

$H(t)$ : Heaviside function (1 for  $t > 0$ , 0 otherwise)

$$\Delta N(t) = N(t) - N(0) = R_L t_a \ln[1 + (R^+/R_L)(\exp(t/t_a) - 1)]$$

or

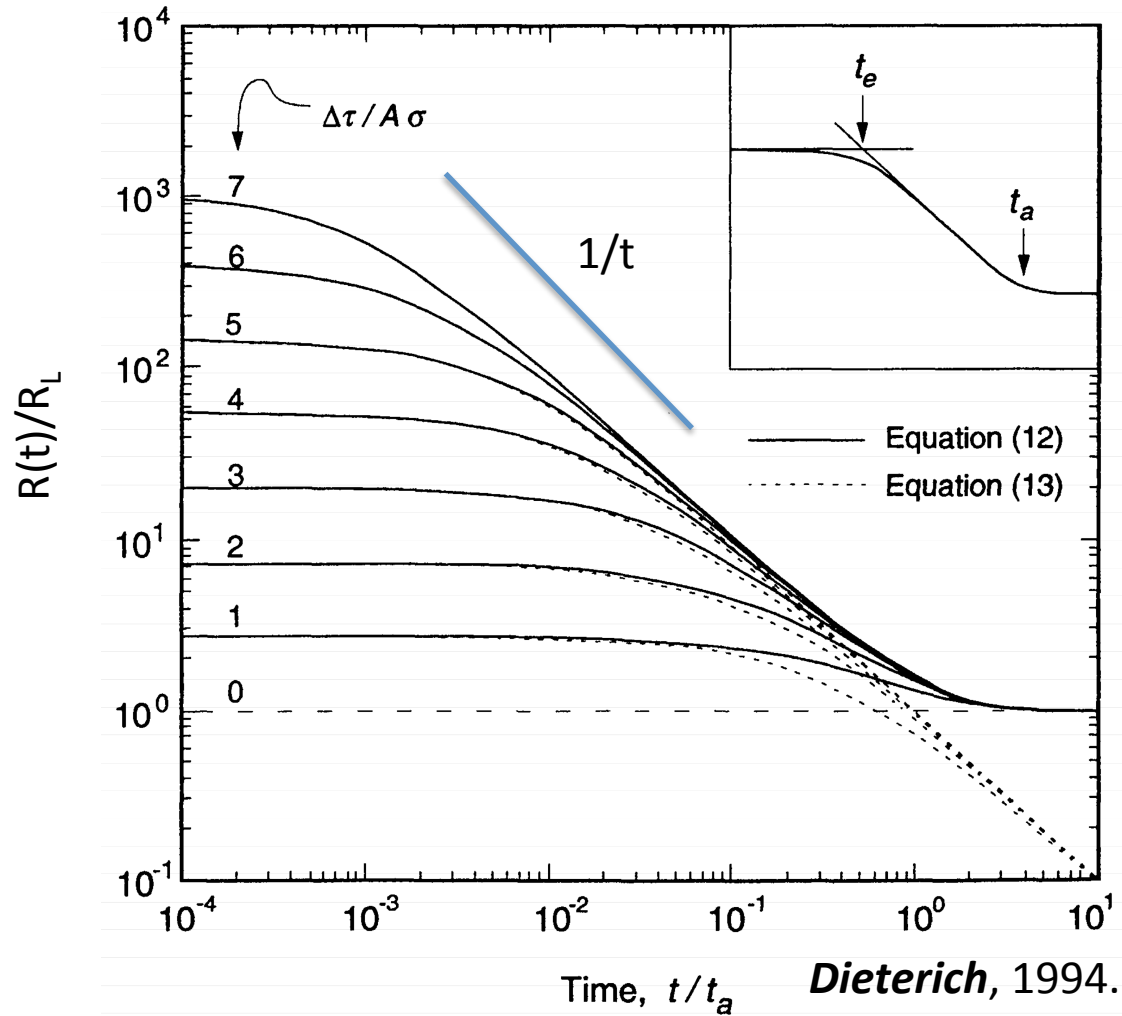
$$R(t) = R^+ \exp(t/t_a) / [1 + (R^+/R_L)(\exp(t/t_a) - 1)]$$

with

$R^+ = R(0) \exp(\Delta\tau_{\text{cos}}/A)$ : Seismicity rate right at the end of the coseismic phase

# Application to a coseismic step

$$R(t) = R^+ \exp(t/t_a) / [1 + (R^+/R_L)(\exp(t/t_a) - 1)]$$

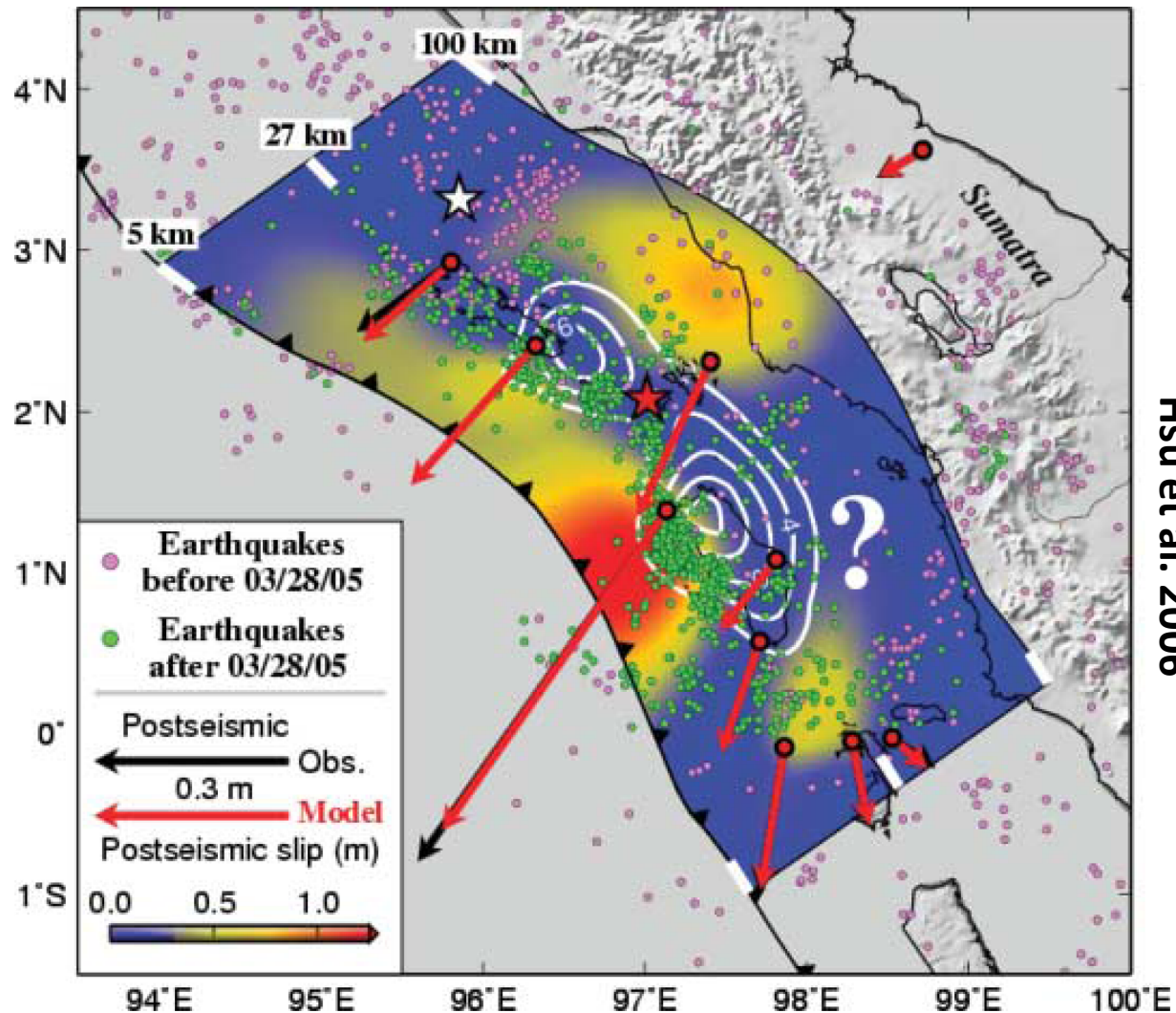


- Seismicity rate jumps from  $R_L$  to  $R^+$  at the time of the mainshock
- Decay as  $1/\text{time}$
- Omori law with an exponent of 1

# **Seismicity Model Based on Afterslip**

# Examples of Afterslip

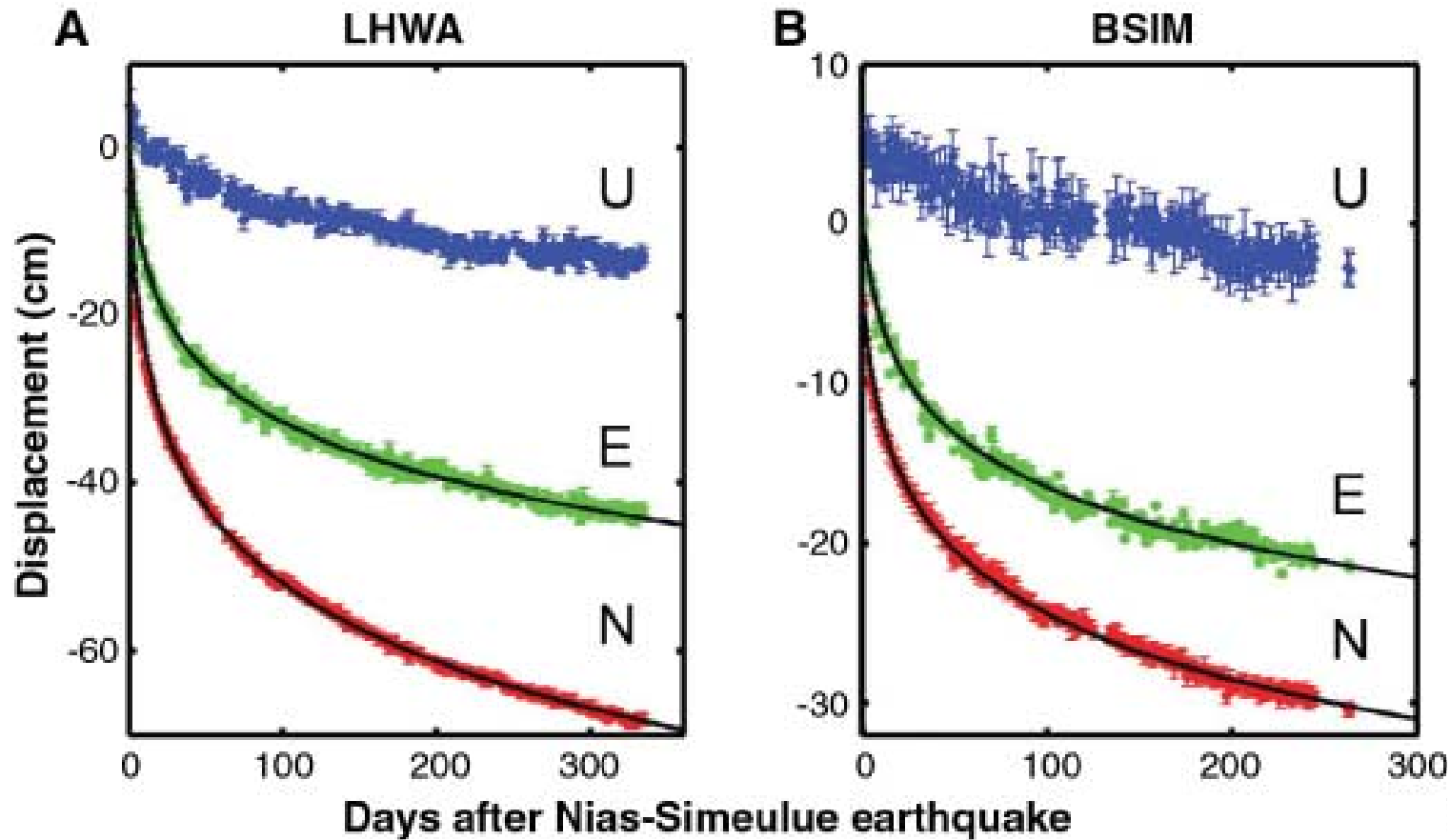
# After slip of Nias (2005, $M_w=8.7$ )



Hsu et al. 2006

- Anti-correlation between co- and postseismic slip
- Aftershocks occurs at the transition between co- and postseismic slip

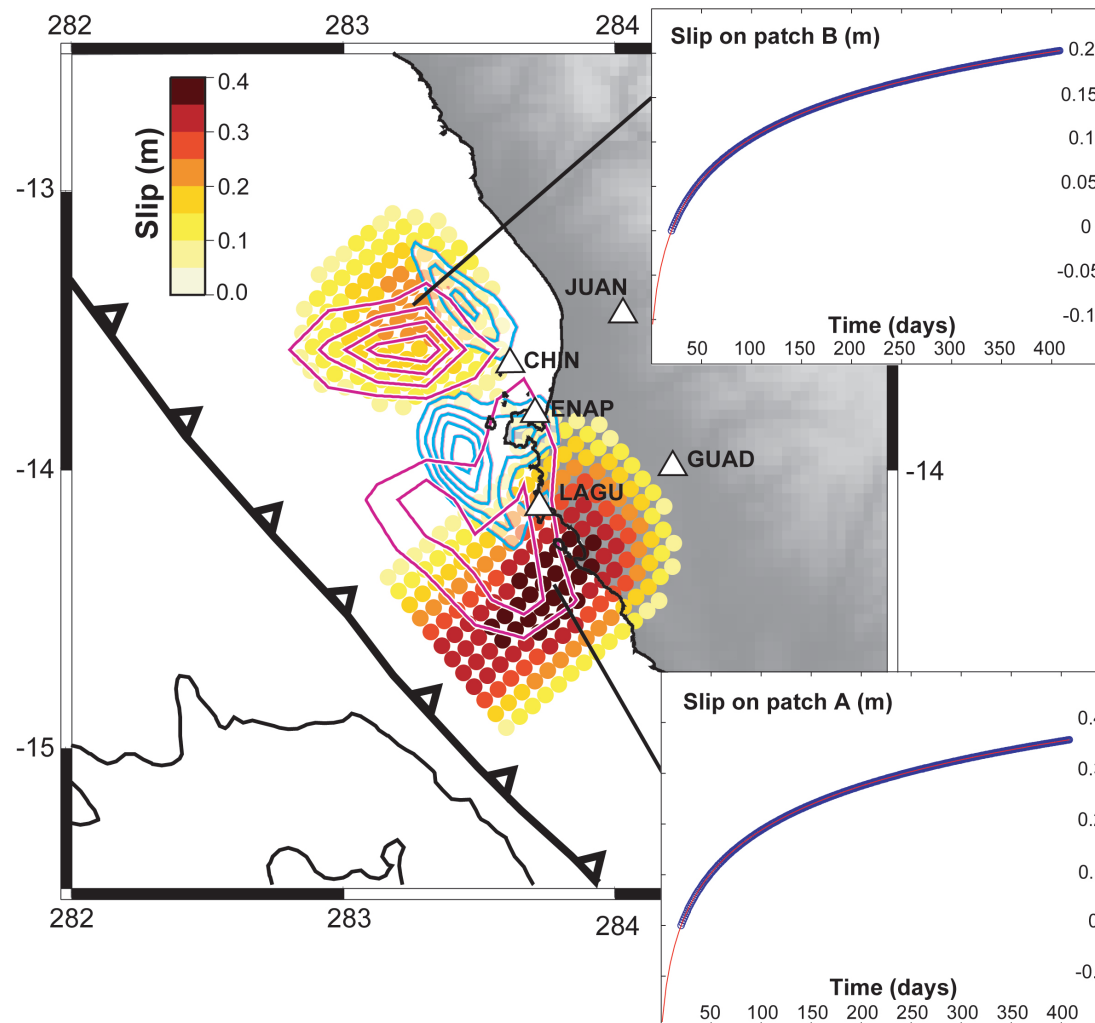
# After slip of Nias (2005, $M_w=8.7$ )



Hsu et al. 2006

After slip grows as the  $\log(\text{time})$

# The Pisco Earthquake (2007, $M_w$ 8.0)

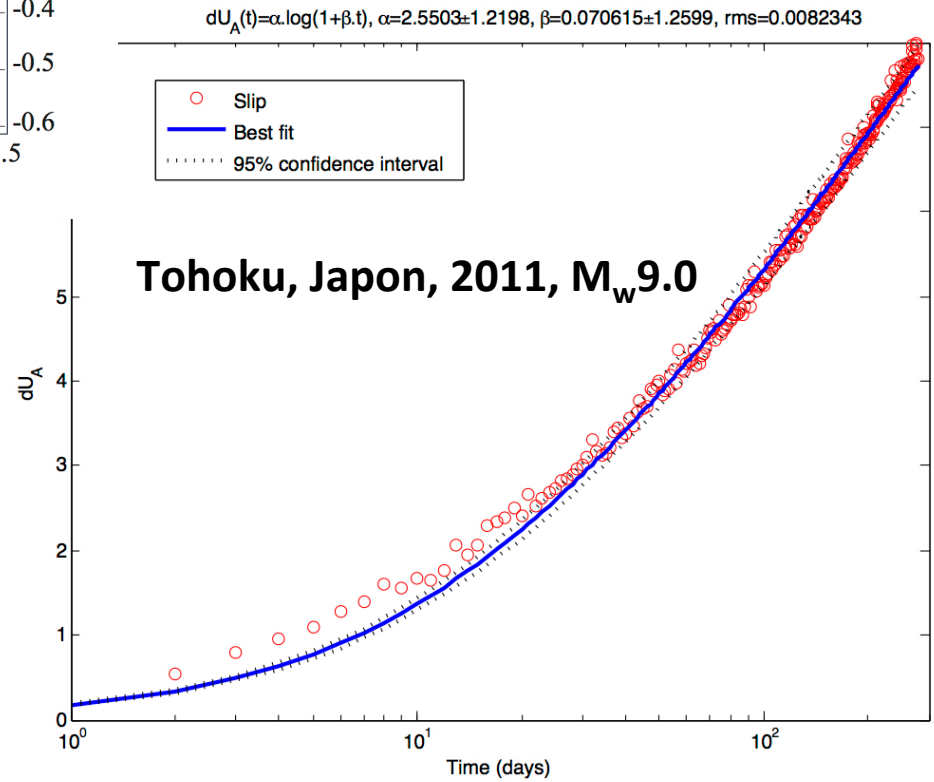
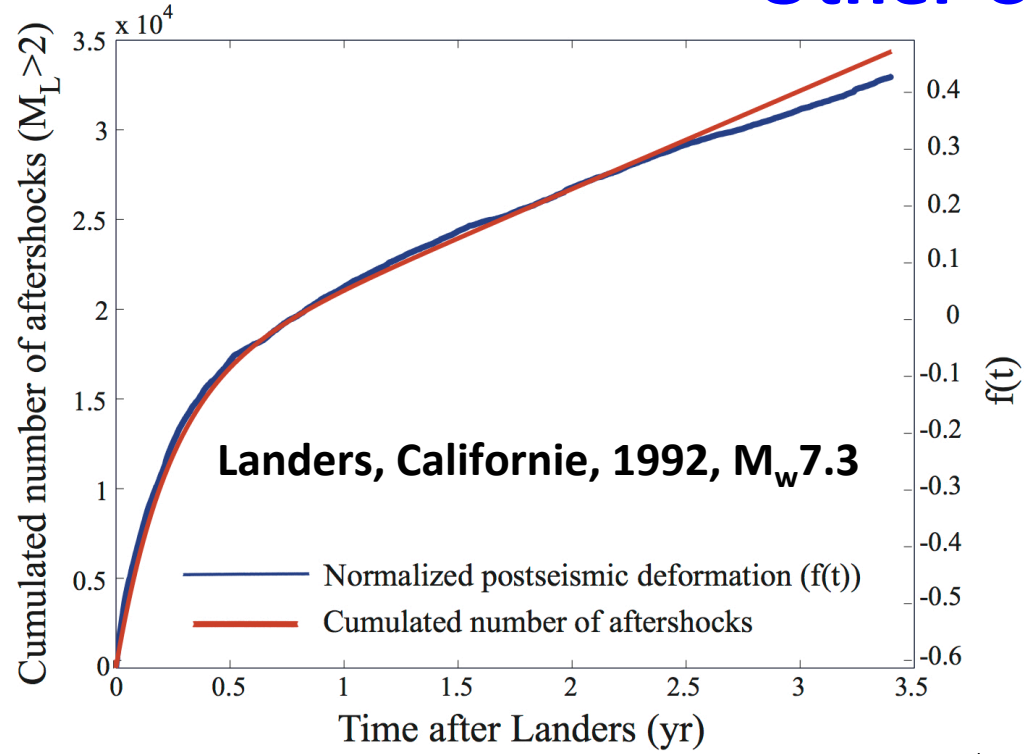


Perfettini et al. 2010

- Afterslip grows as the log(time)
- The distribution of seismicity follows the distribution of afterslip

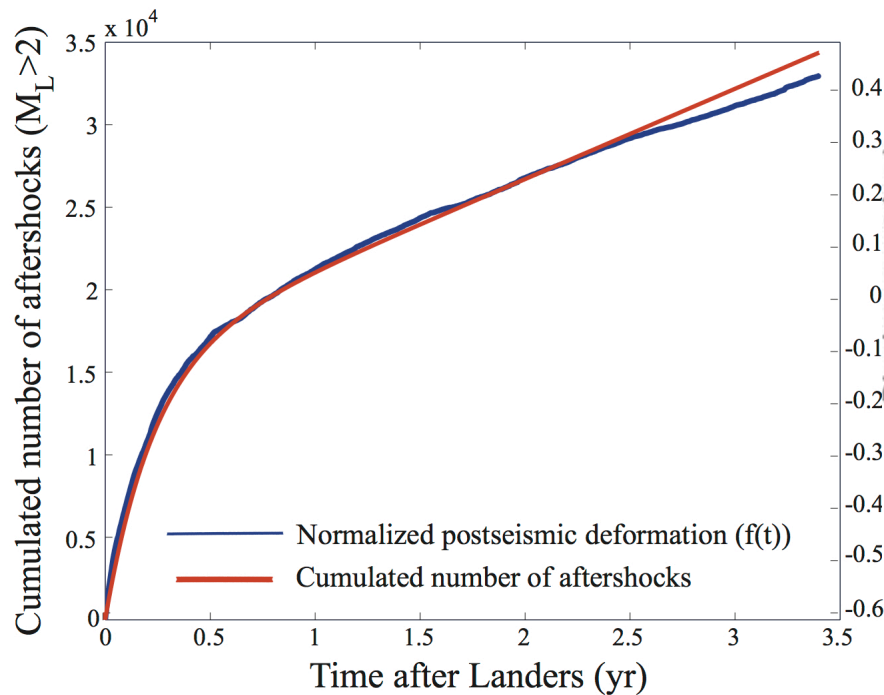


# Other Cases

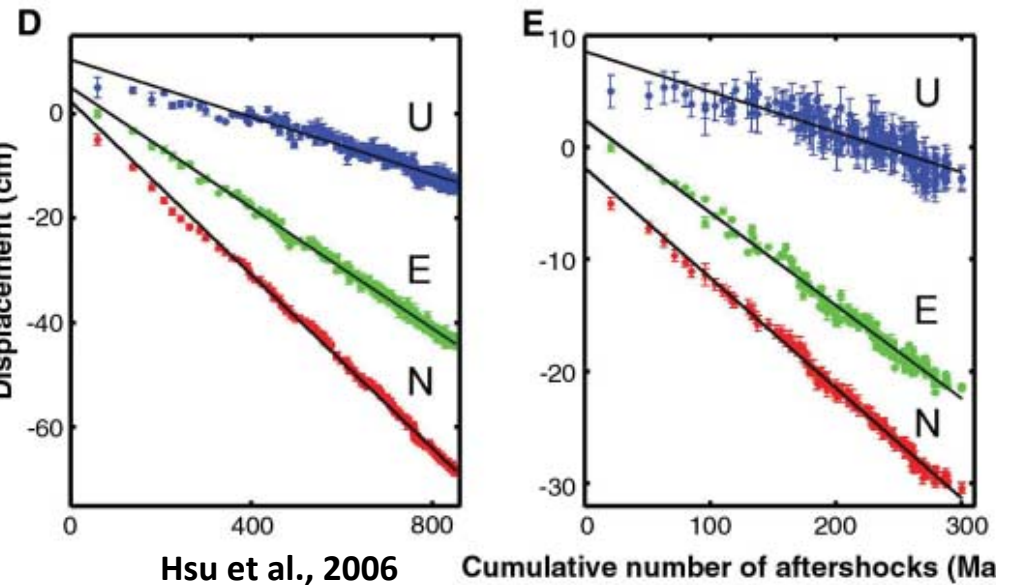


# Link Between Afterslip and Seismicity

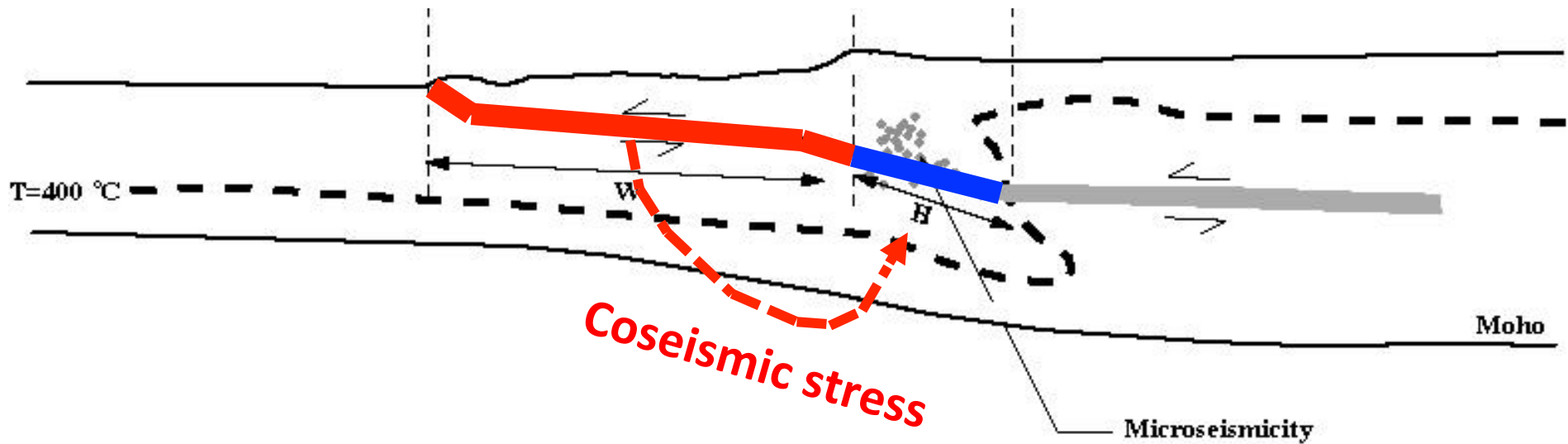
- Afterslip varies as the log(time)  $\rightarrow$  Slip rate  $\sim 1/\text{time}$
- Seismicity rate varies as  $1/\text{time}$  (Omori law)
- Afterslip and seismicity seems to have the same temporal evolution (ex: Chi-Chi, Landers, Nias,...)



Perfettini and Avouac, 2007



# Response of a rate strengthening region to a stress step

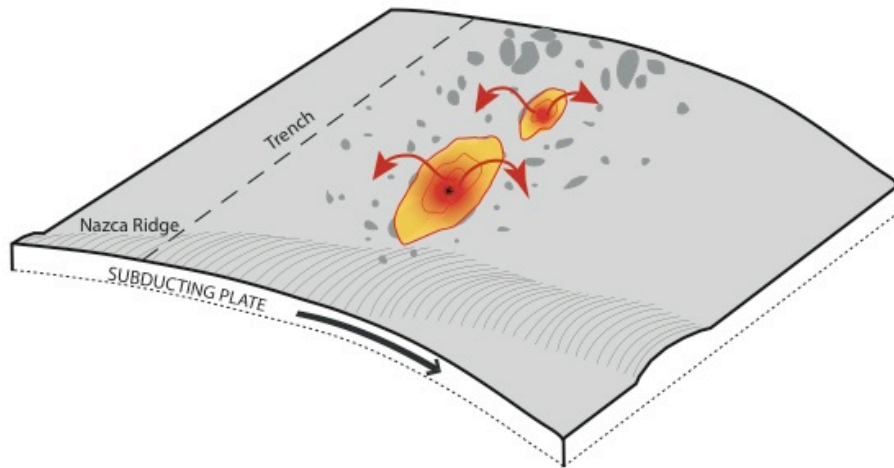




$$\text{Frictional stress: } \tau_f = \sigma [\mu_* + (a-b) \ln(V/V_*)]$$

**$a-b > 0$** : rate strengthening friction

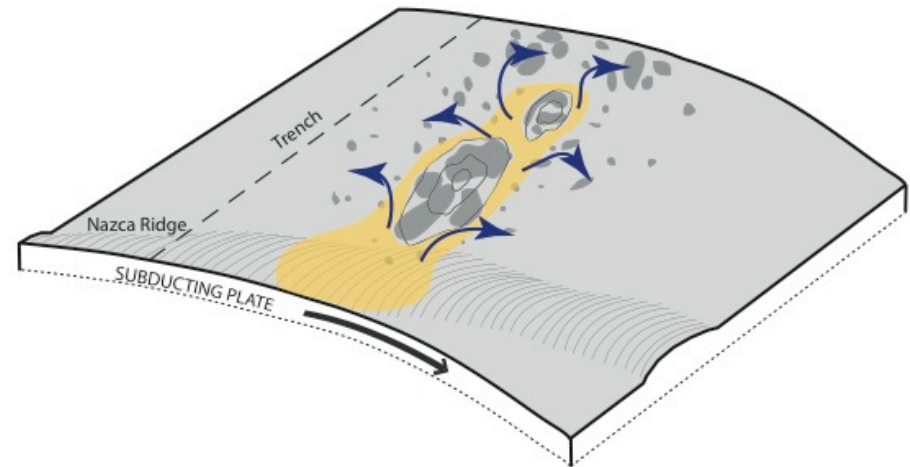
# Response of a rate strengthening region to a stress step

## Coseismic slip



-  Coseismic stress transfer
-  Coseismic slip

## Afterslip



-  Postseismic stress transfer
-  Afterslip

$$\text{Frictional stress: } \tau_f = \sigma [\mu_* + (a-b) \ln(V/V_*)]$$

**$a-b > 0$** : rate strengthening friction

# Afterslip Model

$$\text{slip}_{\text{post}}(t) = V_L t_r \log[1 + (V^+/V_L) \times (\exp(t/t_r) - 1)]$$

$$V_{\text{post}}(t) = V^+ \exp(t/t_r) / [1 + (V^+/V_L) \times (\exp(t/t_r) - 1)]$$

- $V_L$ : long term sliding velocity
- $V^+$ : sliding velocity at the end of the coseismic phase
  - $t_r$ : relaxation time of the postseismic phase
    - $t_r = (a-b)\sigma / (d\tau_L dt)$
- $a-b > 0$ : parameters of the rate and state friction law
  - $\sigma$ : effective normal stress
  - $d\tau_L dt$ : long term stressing rate
    - $V^+ = V_L \times \exp(\Delta\tau / (a-b)\sigma)$
  - $\Delta\tau$ : coseismic shear stress step

The law:  $\text{slip}_{\text{post}}(t) = V_L t_r \log[1 + (V^+/V_L) \times (\exp(t/t_r) - 1)]$   
can adjust postseismic slip following recent earthquake sequences (Chi-Chi, Arequipa, Nias, Pisco, Maule, Tohoku,...)

The parameter  $(a-b)\sigma$  is not varying too much with  $(a-b)\sigma = 1-10$  bars, suggesting either a low effective normal stress (high pore pressure?) and/or that the area of high postseismic response are close stability ( $a-b$  near 0)

# Seismicity Model based on Afterslip

- We assume that the seismicity rate  $R(t)$  is proportional to the deformation rate, itself proportional to the afterslip rate:

$$R(t) = dN_{\text{cum}}/dt = c(t) V_{\text{post}}(t)$$

$$N_{\text{cum}}(t) = \left[ C(t) U_{\text{post}}(t) - C(0) U_{\text{post}}(0) \right] - \int_0^t U_{\text{post}}(t') [dc(t')/dt'] dt'$$

Since  $U_{\text{post}}(0) = 0$ :

$$N_{\text{cum}}(t) = c(t) U_{\text{post}}(t) - \int_0^t U_{\text{post}}(t') [dc(t')/dt'] dt'$$

We further assume that  $c(t) = \text{cste}$  ( $dc/dt = 0$ ):

$$N_{\text{cum}}(t) = c U_{\text{post}}(t)$$

# Seismicity Model based on Afterslip

Seismicity rate given by:

$$R(t) = R^+ \exp(t/t_r) / [1 + (R^+/R_L) \times (\exp(t/t_r) - 1)]$$

$R_L$ : long term seismicity rate

$R^+$ : seismicity rate at the end of the coseismic phase

$t_r$ : relaxation time of afterslip

$$t_r = (a-b)\sigma / (d\tau_L dt)$$

$a-b > 0$ : parameters of the rate and state friction law

$\sigma$ : effective normal stress

$d\tau_L dt$ : loading rate

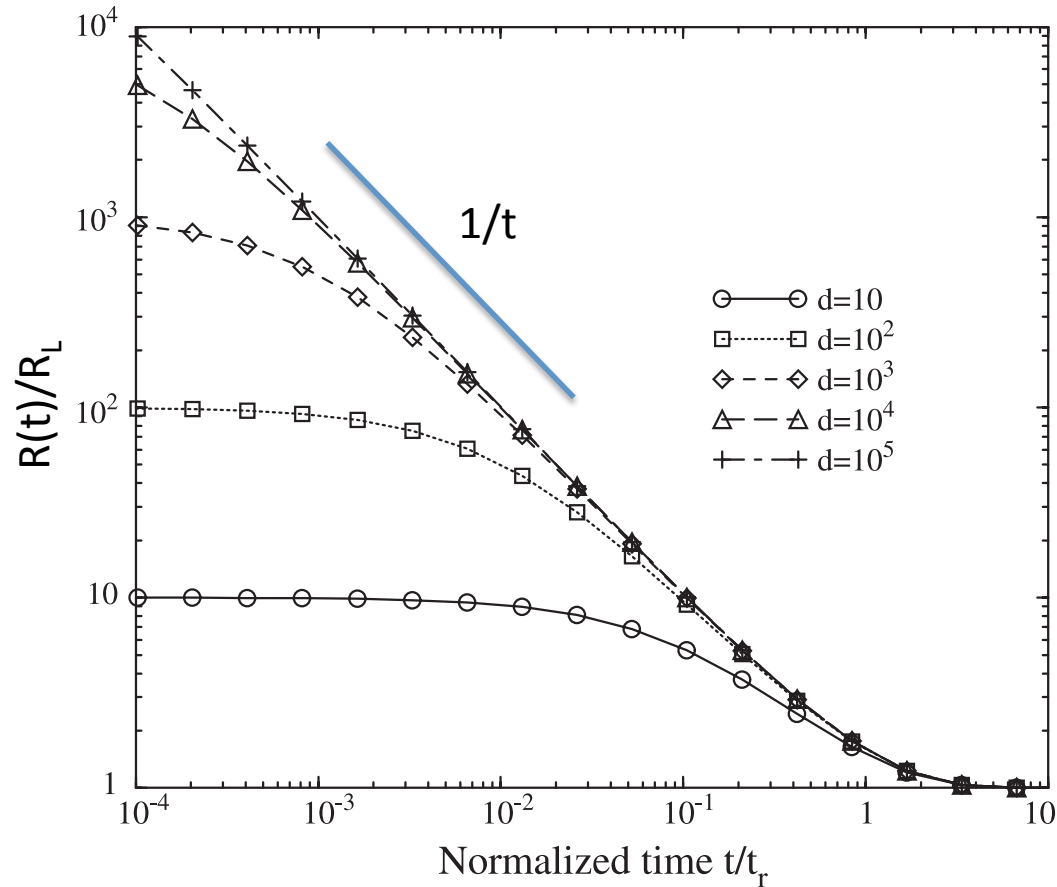
$$R^+ = R_L \times \exp(\Delta\tau / (a-b)\sigma)$$

$\Delta\tau$ : shear stress change



# Seismicity Model

$$R(t) = R^+ \exp(t/t_r) / [1 + (R^+/R_L)(\exp(t/t_r) - 1)]$$



- Seismicity rate jumps from  $R_L$  to  $R^+$  at the time of the mainshock
- Decay as **1/time**
- Omori law with an exponent of 1

*Perfettini and Avouac, 2004.*

# Afterslip Model vs. Dieterich's Model

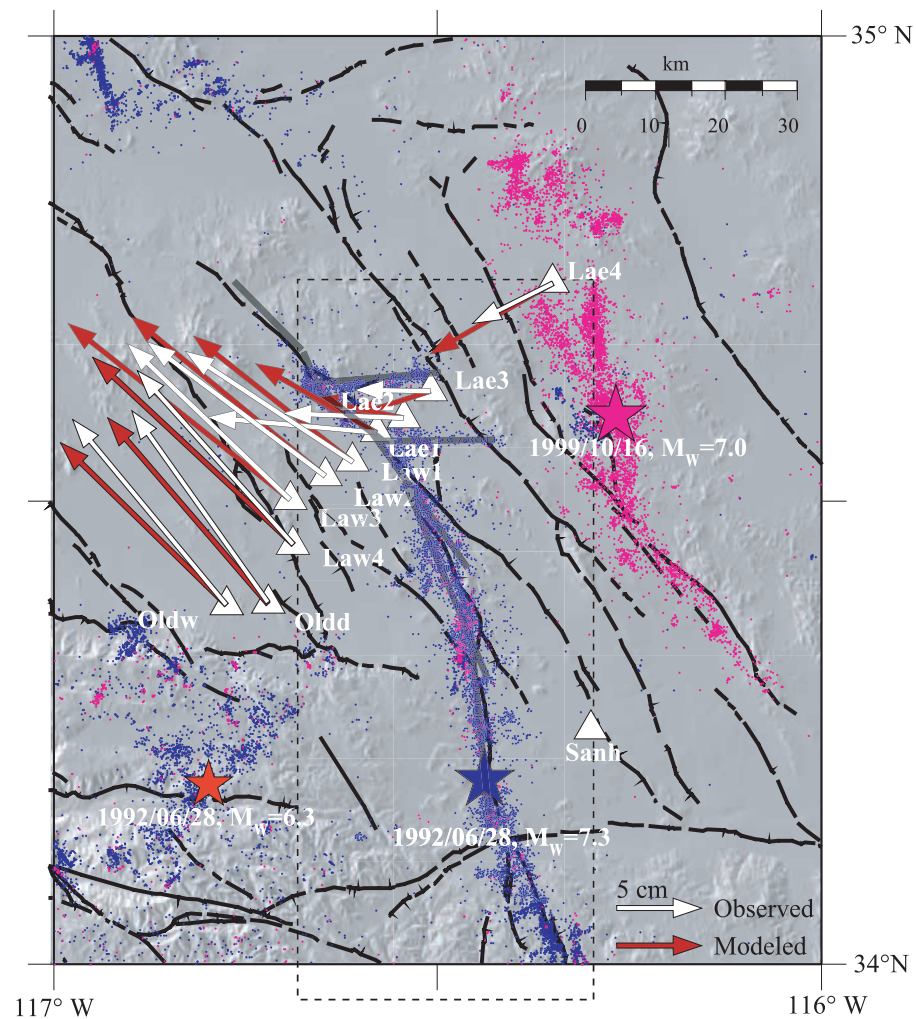
**Afterslip:**  $R(t) = R^+ \exp(t/t_r) / [1 + (R^+/R_L)(\exp(t/t_r) - 1)]$

**Dieterich:**  $R(t) = R^+ \exp(t/t_a) / [1 + (R^+/R_L)(\exp(t/t_a) - 1)]$

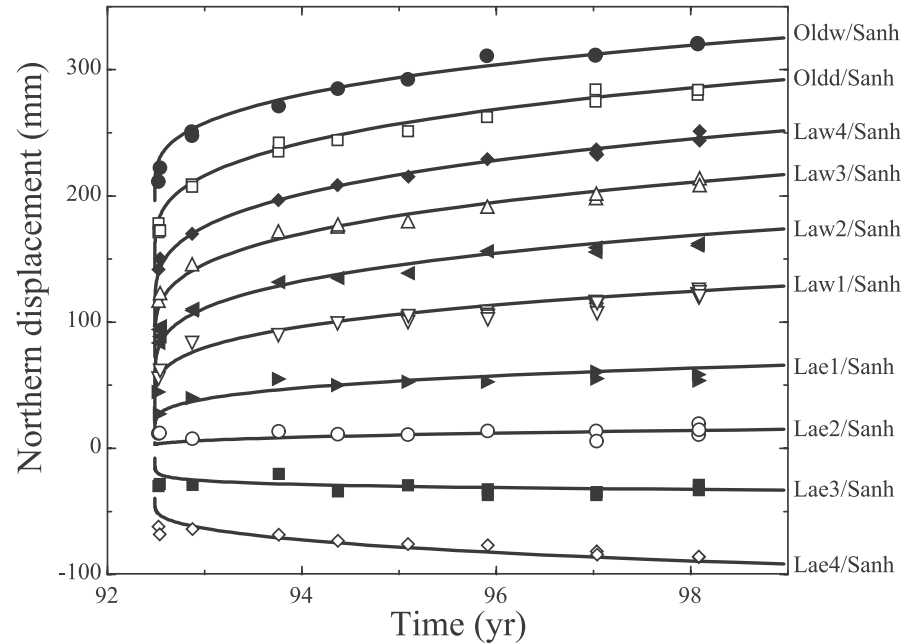
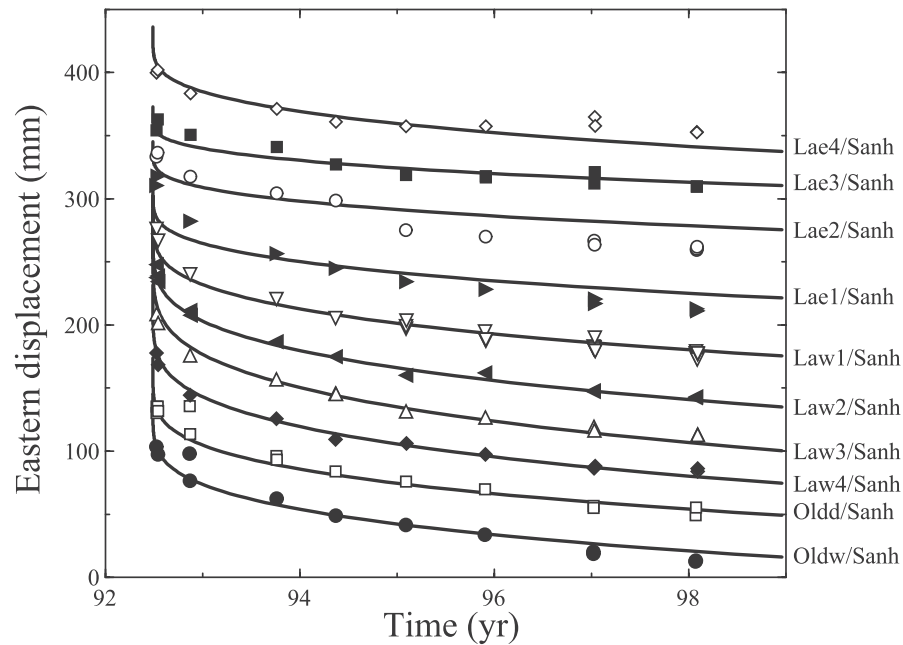
- Both models are identical mathematically
- The variable  $\gamma(\mathbf{t})$  of Dieterich's model is proportional to  $1/V_{\text{post}}(\mathbf{t})$  in the afterslip seismicity model
- The assumption of a constant seismicity rate in the afterslip model arises naturally because the creeping zone relaxes to the long term velocity
- The afterslip model has the additional constrain that  $R(\mathbf{t})$  is proportional to the afterslip velocity  $V_{\text{post}}(\mathbf{t})$

# **Case of the Landers Earthquake**

# Postseismic GPS data



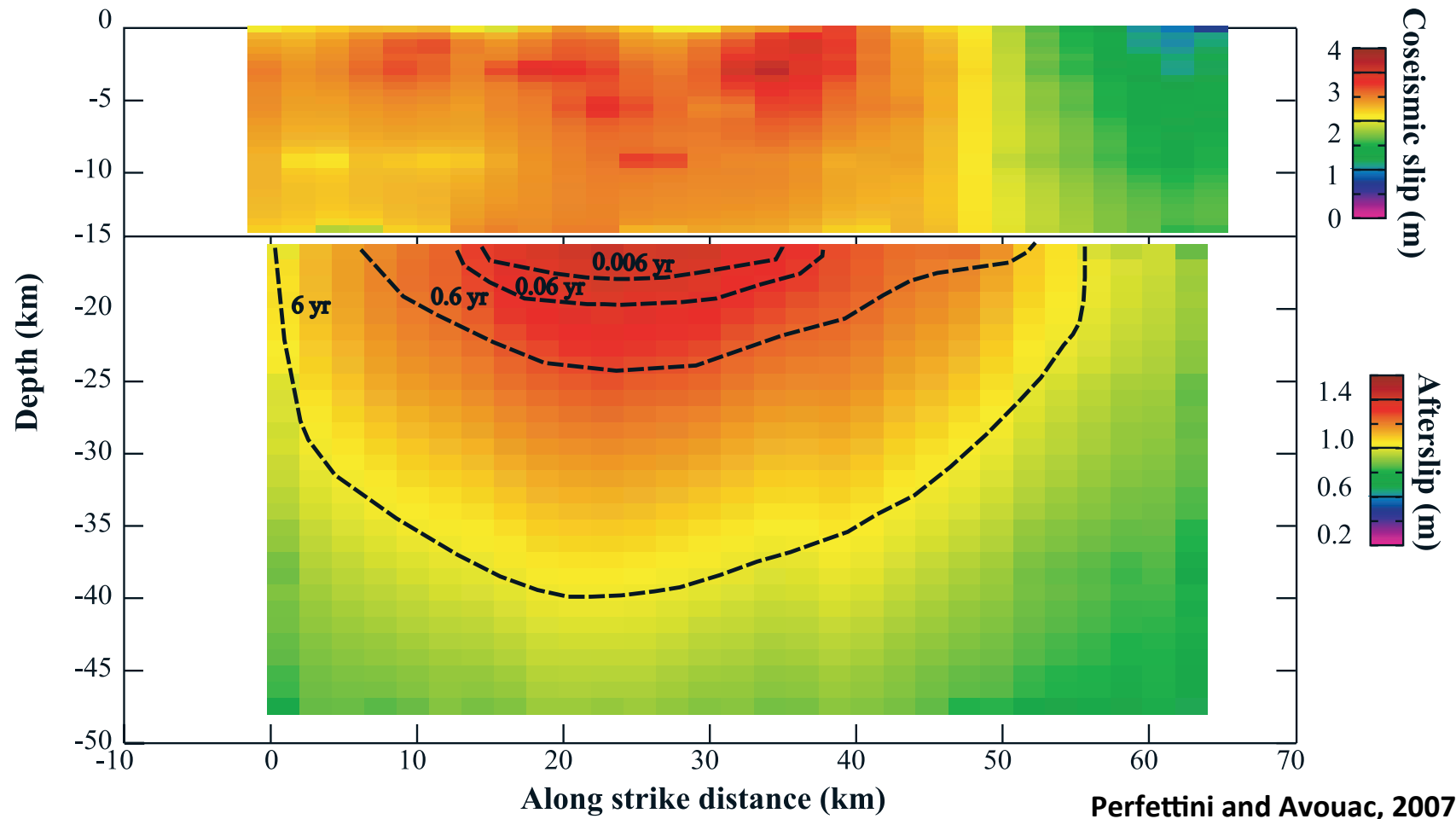
# Postseismic GPS Time Series



Perfettini and Avouac, 2007

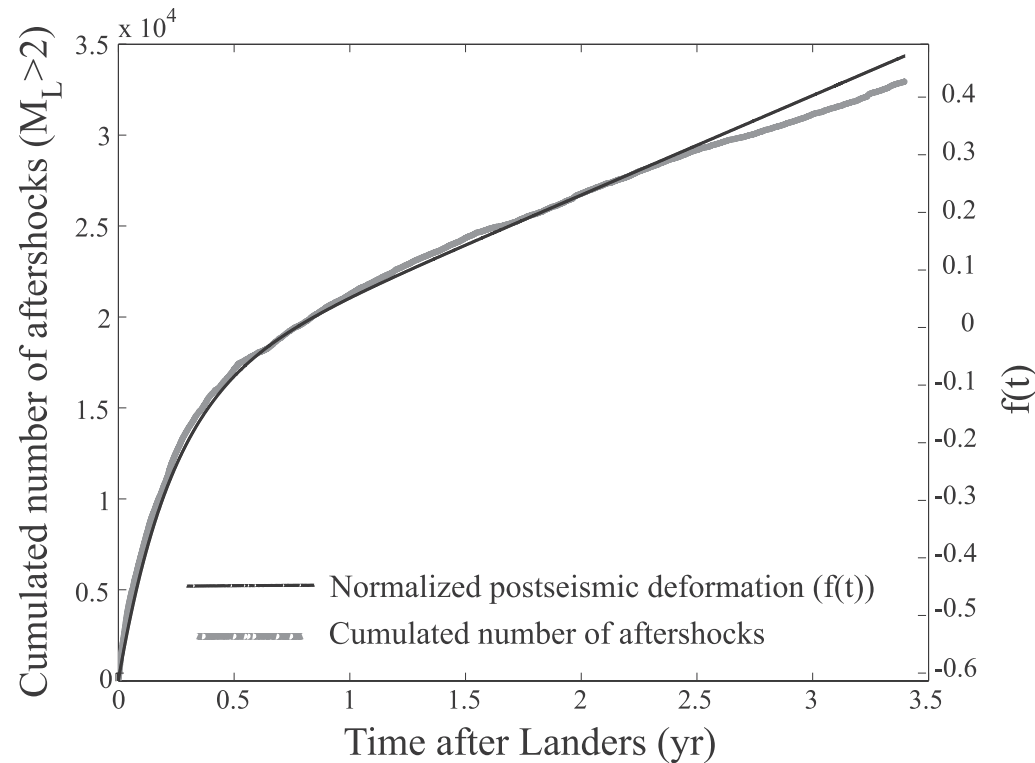
- The inverted afterslip distribution adjusts very well the observed time series
- The afterslip model is a quasi-dynamic model based on rate and state friction on a rate strengthening fault

# Afterslip Model of Landers



- Afterslip is located below the seismogenic zone
- It is spreading with time but the spatial pattern remains the same

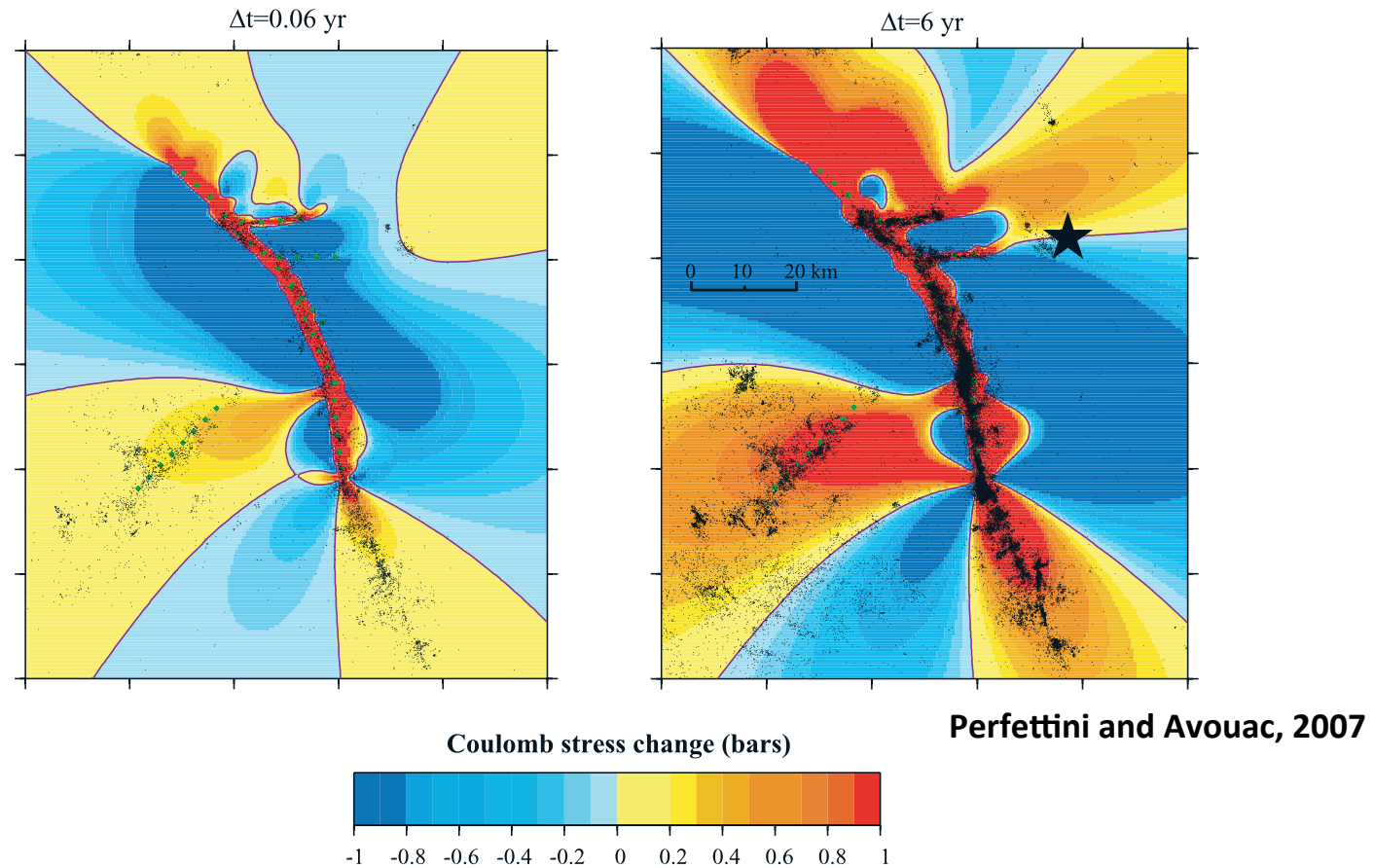
# Seismicity Rate vs. Slip Rate



In the case of the Landers earthquake, the cumulated number of aftershocks is proportional to the postseismic deformation inferred by GPS:

$$N_{\text{cum}}(\mathbf{t}) = c U_{\text{post}}(\mathbf{t})$$

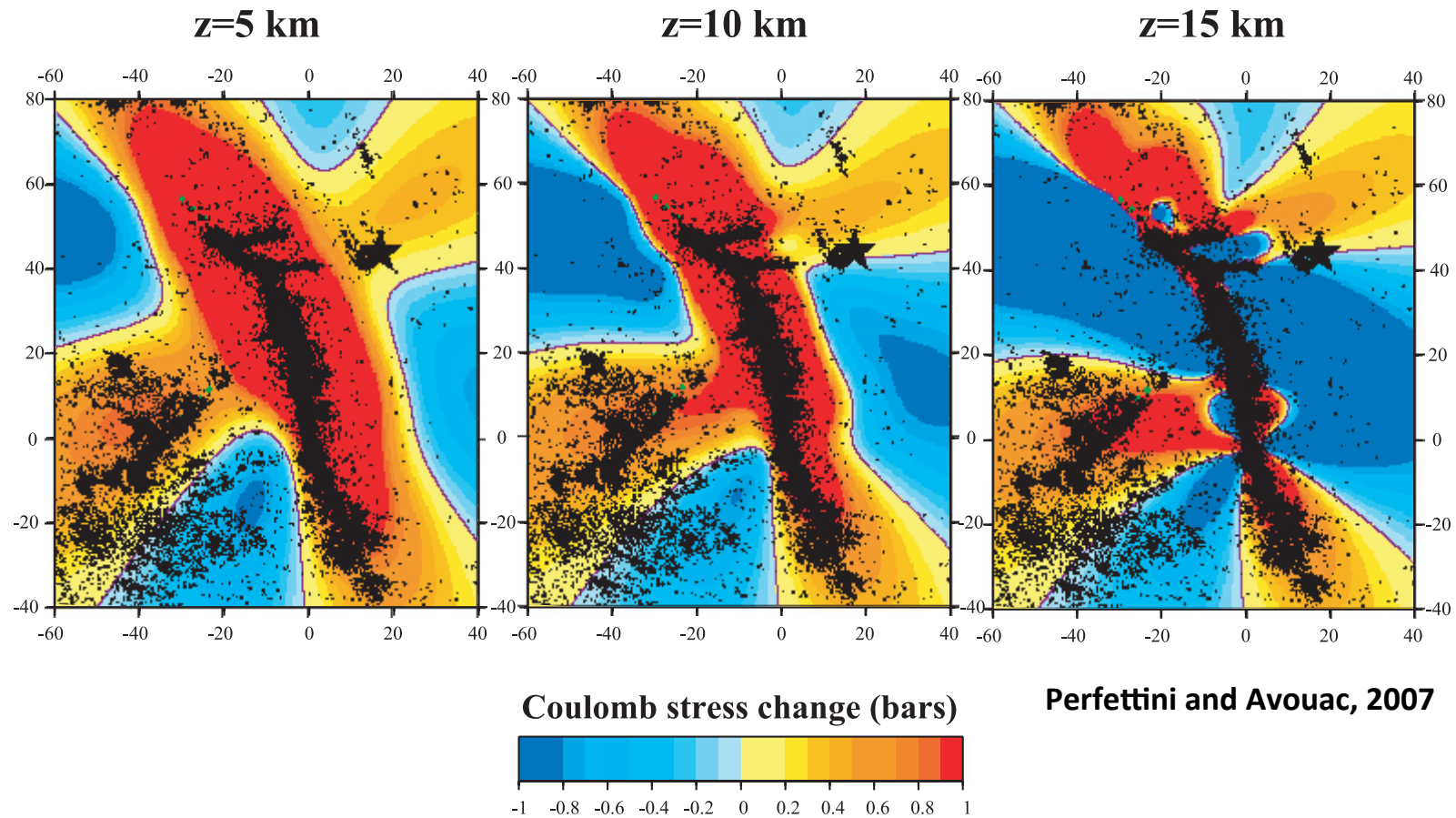
# Postseismic Coulomb Stress at the base of the seismogenic zone



- Postseismic Coulomb stress are always positive near the fault assuming receiver faults oriented as the regional tectonic trend
- Aftershocks are located in area of postseismic Coulomb stress increase

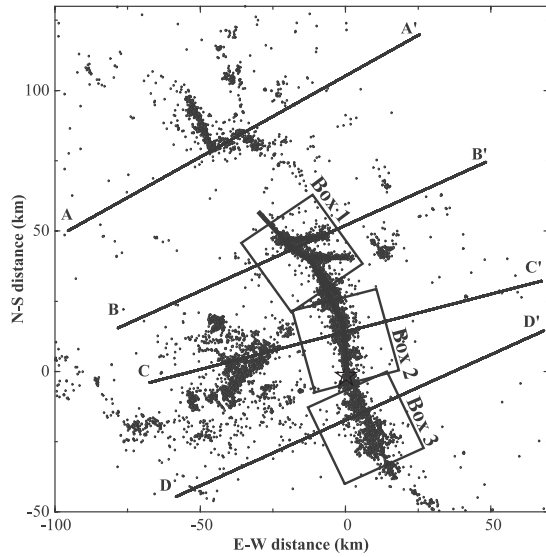


# Postseismic Coulomb Stress

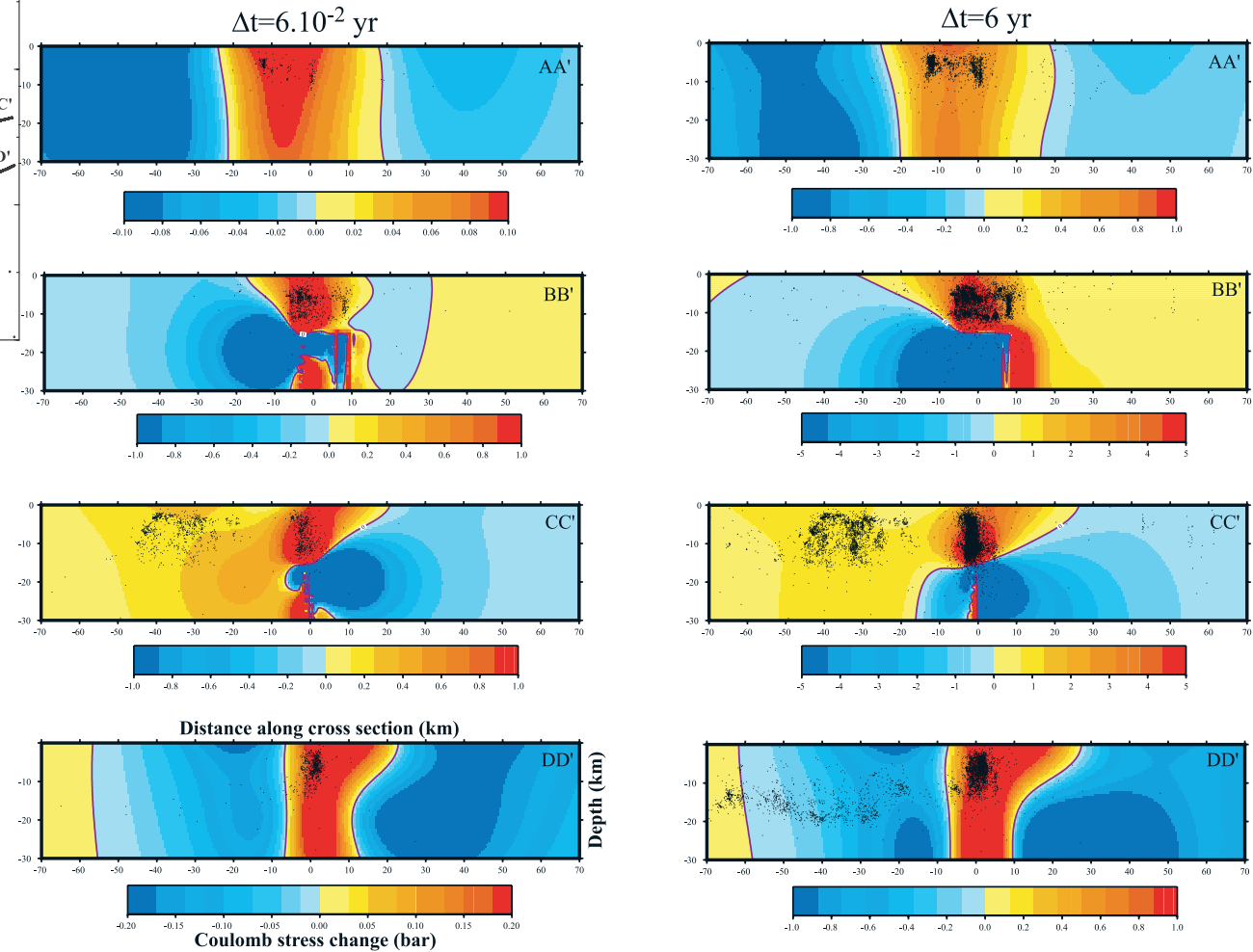


Aftershocks are located in area of increased postseismic stress at all depths

# Postseismic Coulomb Stress: Cross Sections

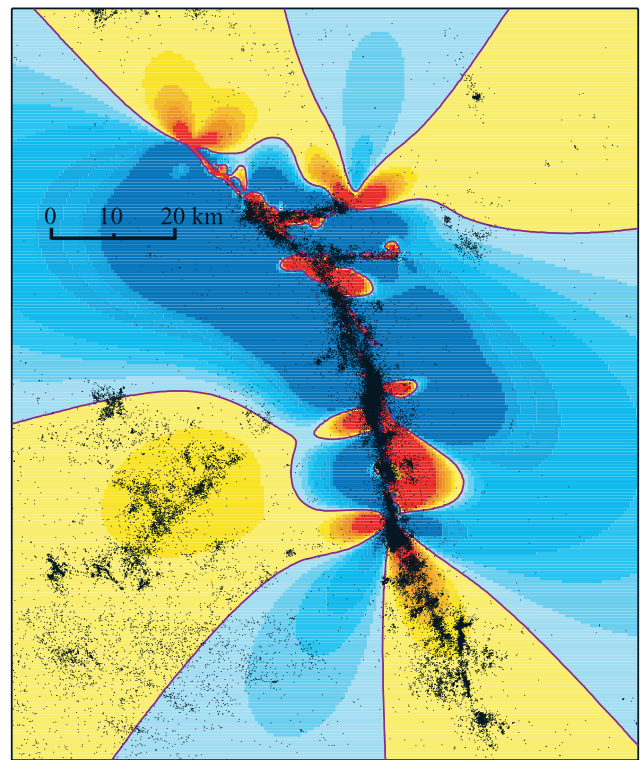


Perfettini and Avouac, 2007

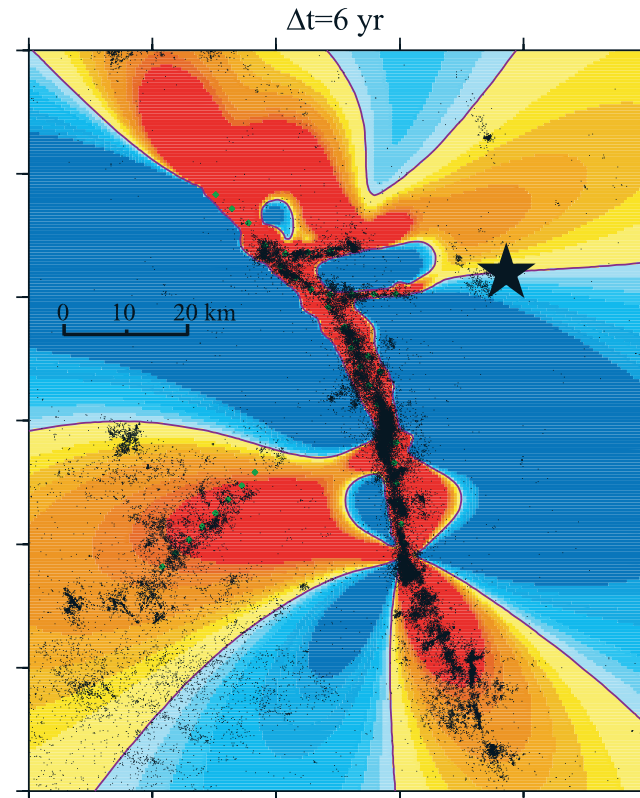
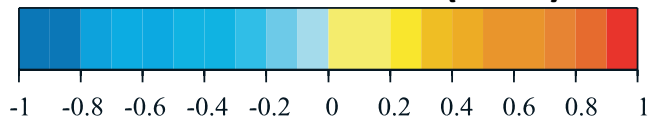


The seismogenic fault zone is always loaded by postseismic stress

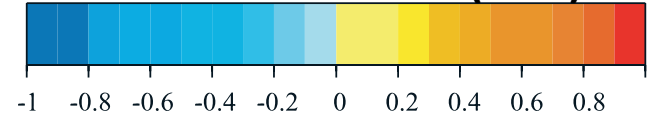
# Coseismic vs. Postseismic Coulomb Stress



$\Delta$ CFF Coseismic (MPa)



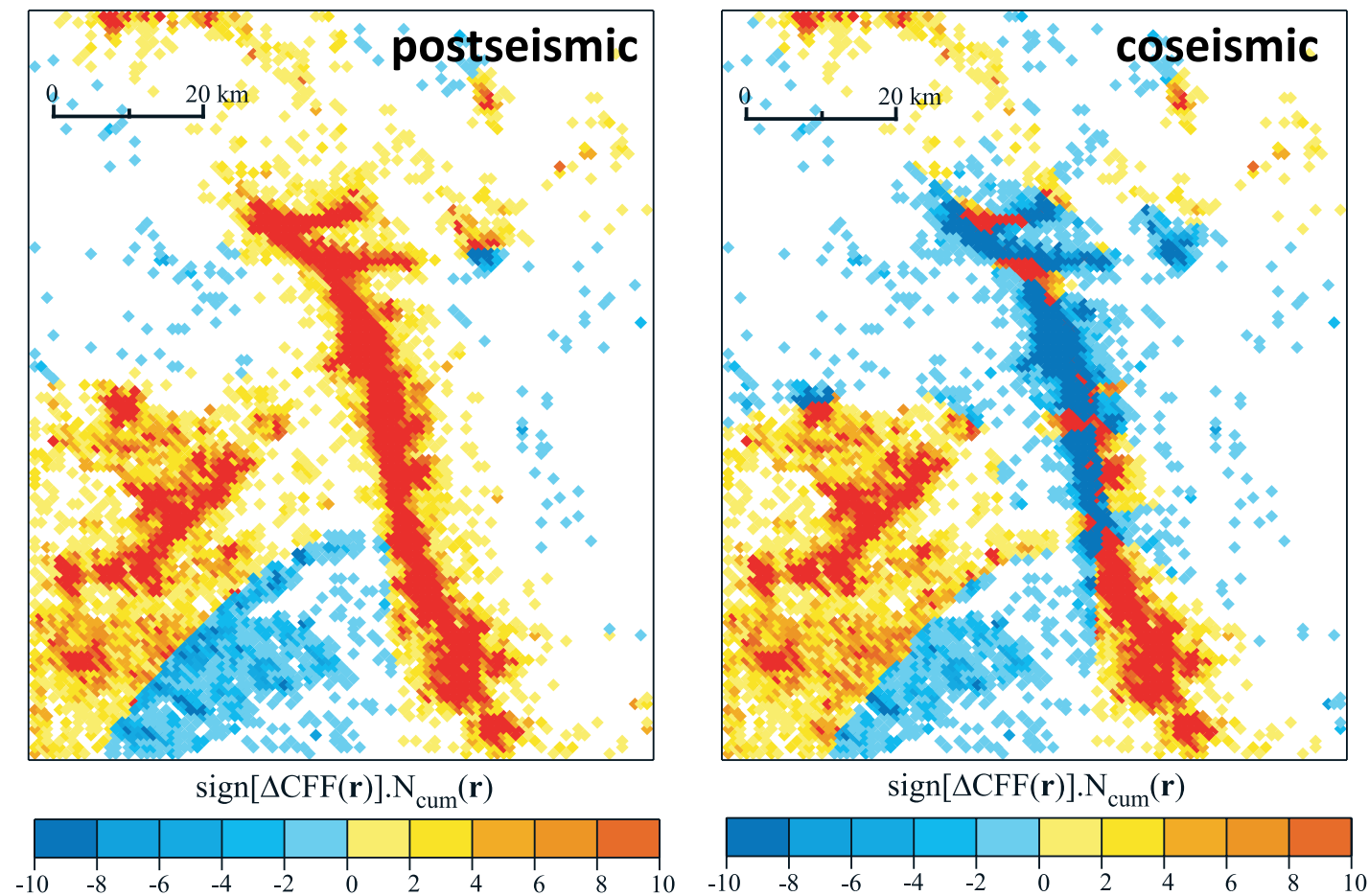
$\Delta$ CFF Postseismic (bars)



Perfettini and Avouac, 2007

- **Coseismic** Coulomb stress changes are mostly **negative** near the rupture plane
- **Postseismic** Coulomb stress changes are **positive** near the rupture plane

# Coseismic vs. Postseismic Coulomb Stress

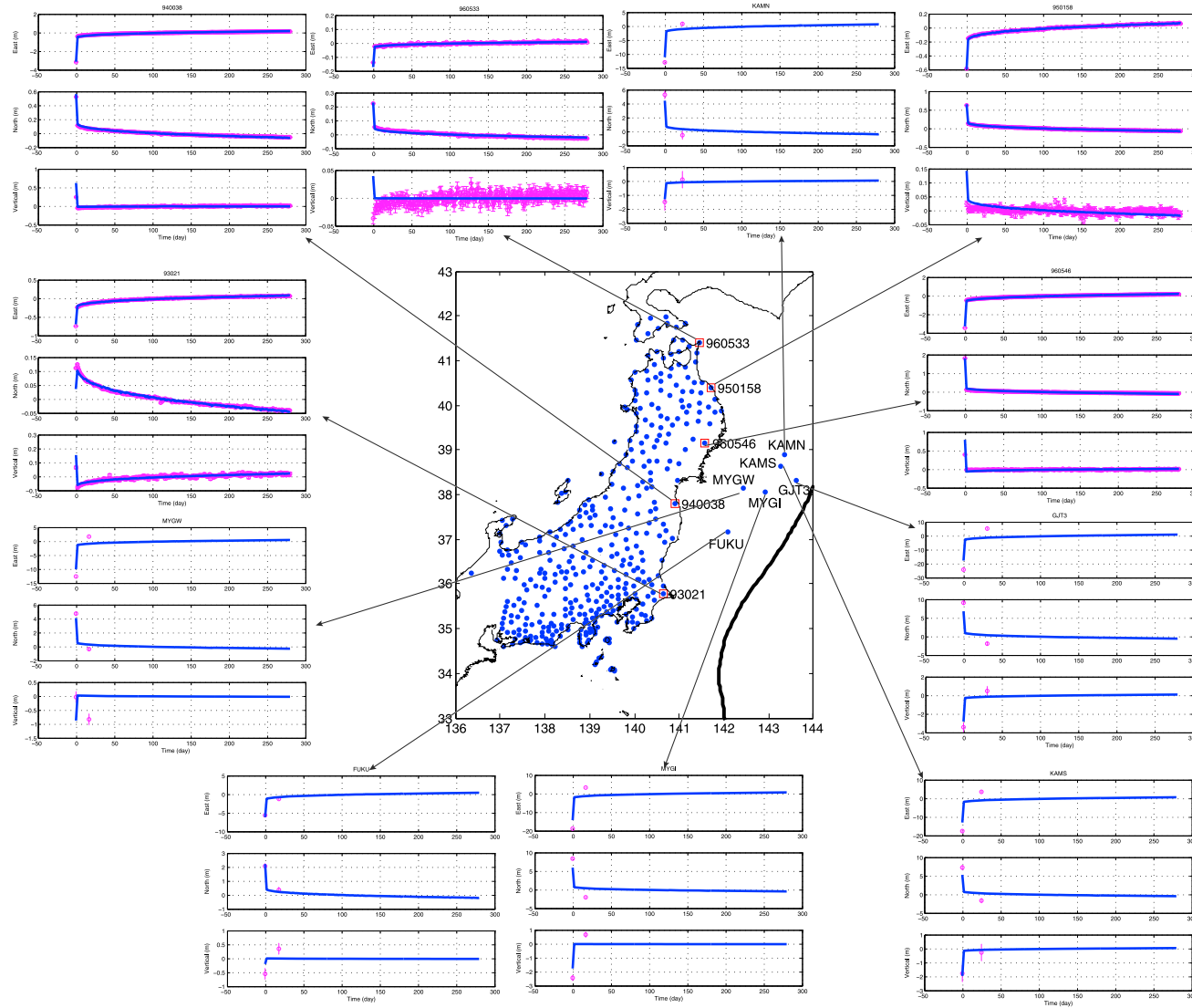


Perfettini and Avouac, 2007

- More than 95% of aftershocks correlated with areas of postseismic Coulomb stress increase
- This amount drops to about 50% when considering coseismic Coulomb stress

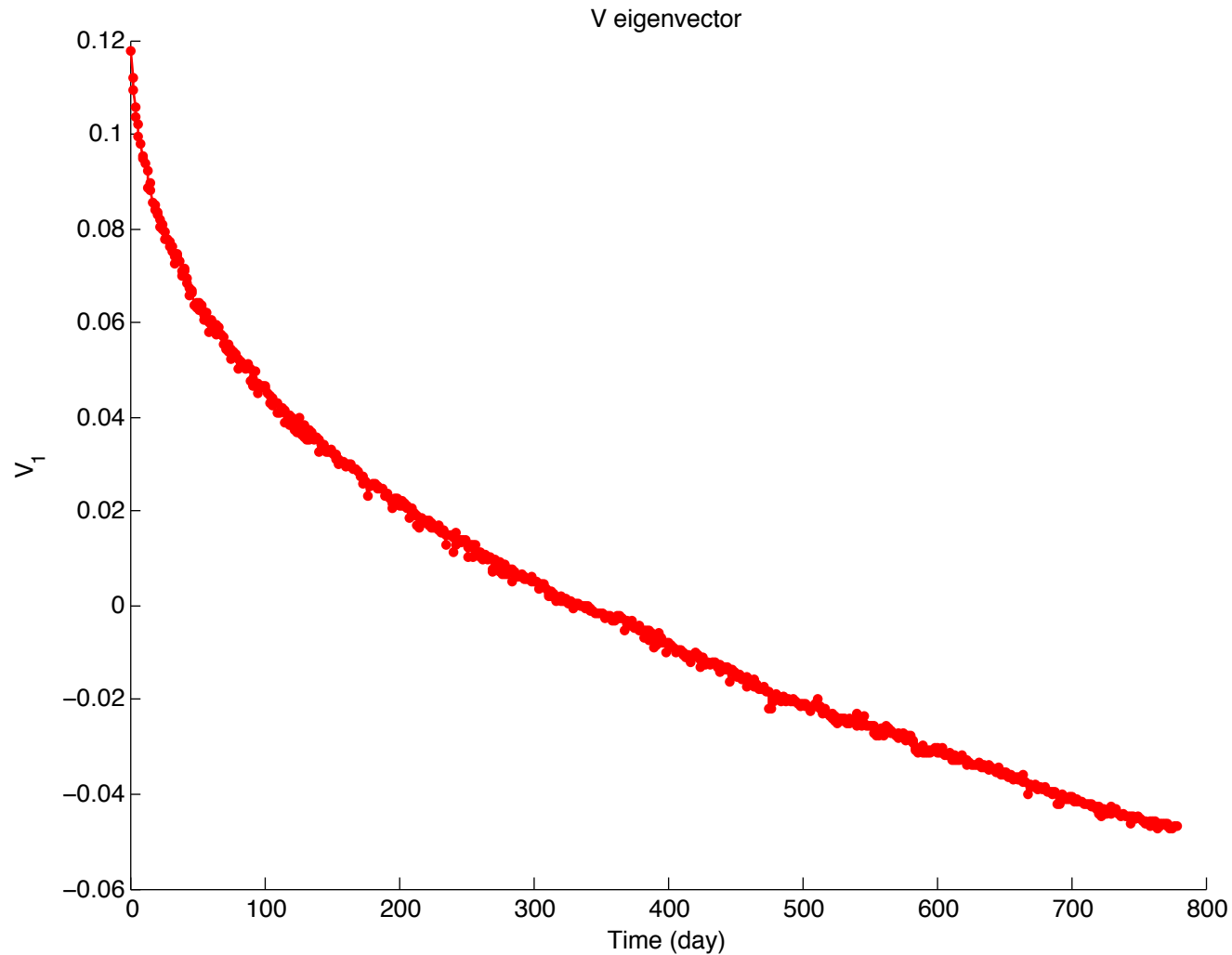
# **Case of the Tohoku-Oki Earthquake**

# Postseismic Deformation



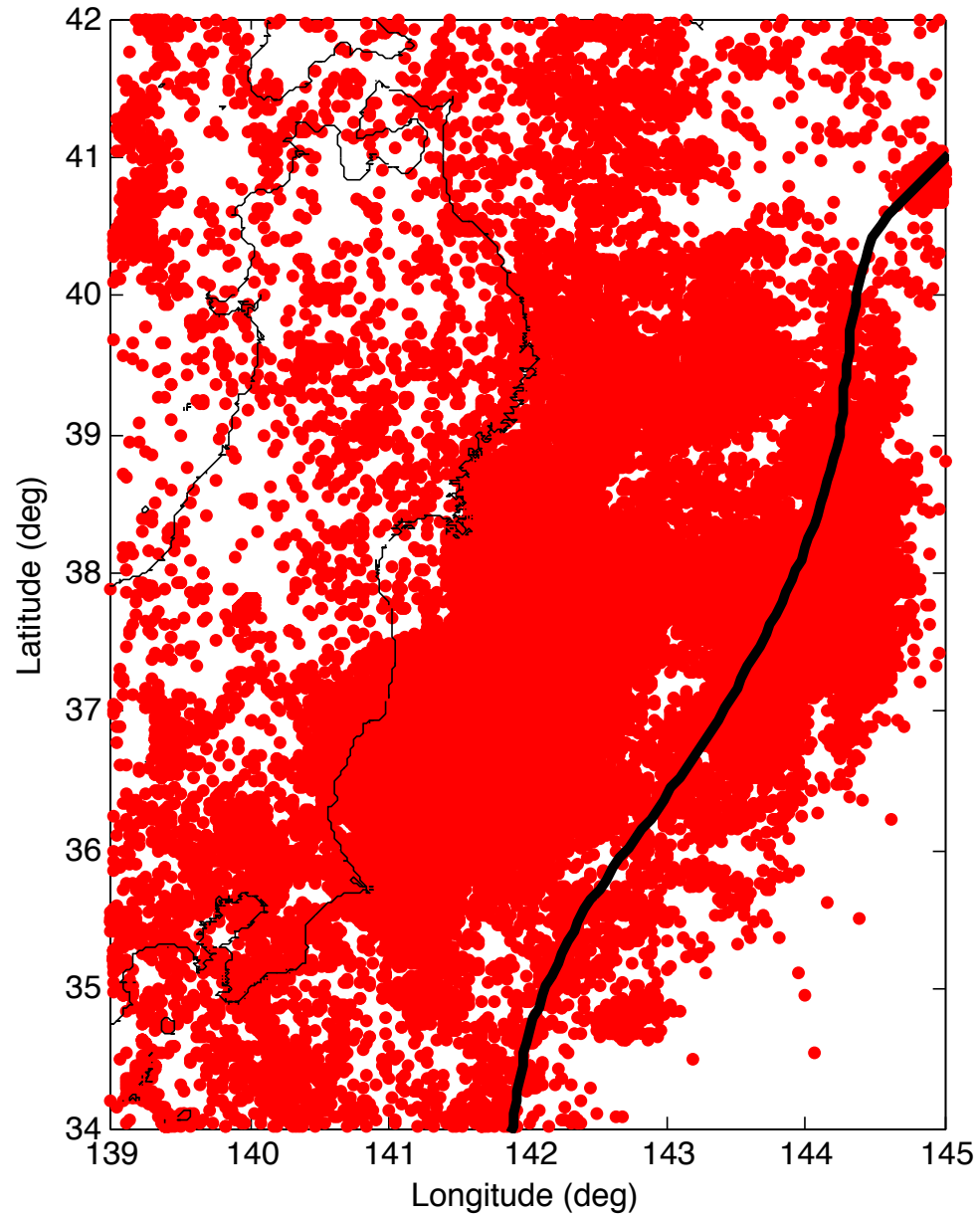
Exceptional set of 401 cGPS stations over 778 days

# Postseismic Deformation: Temporal Evolution



- PCAIM decomposition:  $\mathbf{X}_{\text{dat}} = \mathbf{U}_1 \mathbf{S}_1 \mathbf{V}_1$
- One component alone explains 99.1% of the observations

# Aftershocks



- JMA catalog with  $M_w > 2$
- About 126 000 aftershocks



# Dieterich's Model with Postseismic Stress

$$\Delta N_{\text{cum}}(t) = R_L t_a \ln[1 + (R^+/R_L) J(t)]$$

with

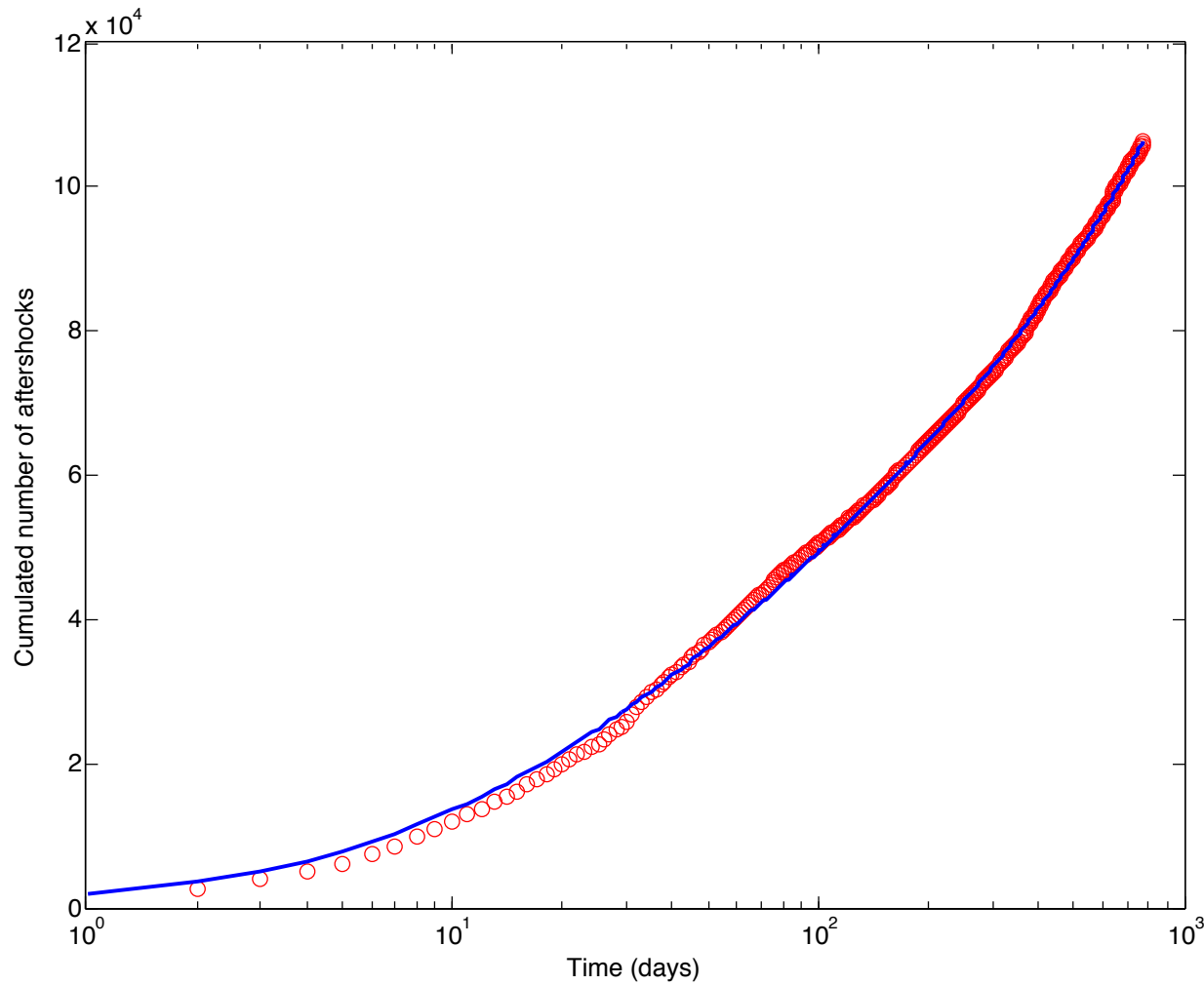
$$R^+ = R(0) \exp(\Delta\tau_{\text{cos}}/A)$$

$$J(t) = (1/t_a) \int_0^t \exp(t'/t_a) \exp[\Delta\tau_{\text{post}}(t')/A] dt'$$

$$\Delta\tau_{\text{post}}(t) = cAV_1(t)$$

$R_L$ ,  $t_a$ ,  $R^+$  and  $c$  are 4 constants to be adjusted

# Dieterich's Model with Postseismic Stress



Very good fit to the data for:

- $R_L=44.46$  evts/day,  $R^+=1906$  evts/day,  $t_a=451.5$  days,  $c=3.586 \cdot 10^{-10}$
- The term involving postseismic stress is not required by the model ( $c \approx 0$ )

# Afterslip Model

$$\Delta N_{\text{cum}}(t) = R_L t_r \ln[1 + (R^+/R_L) J(t)]$$

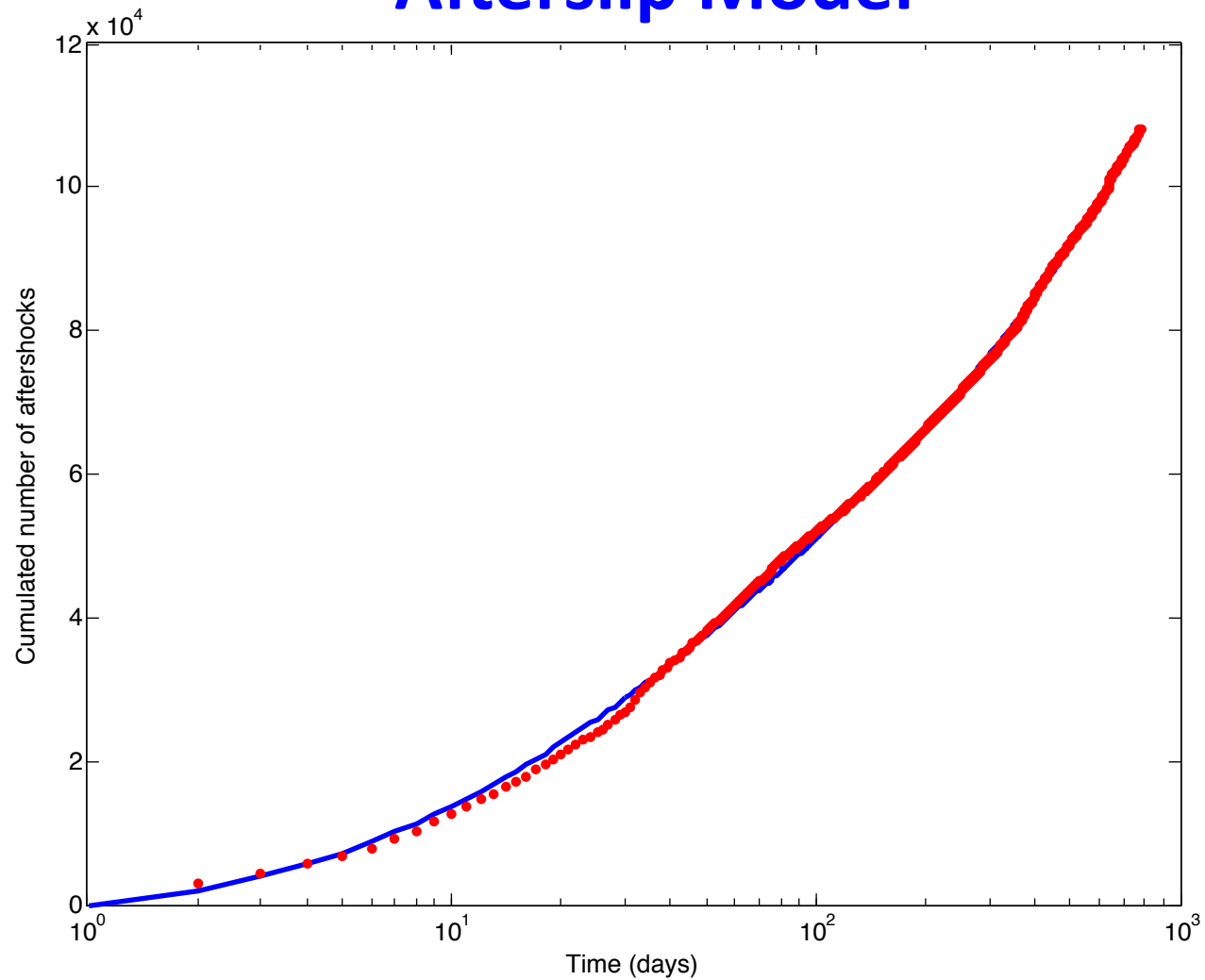
with

$$R^+ = R(0) \exp(\Delta\tau_{\text{cos}}/A)$$

$$J(t) = (1/t_a) \int_0^t dt' \exp(t'/t_a)$$

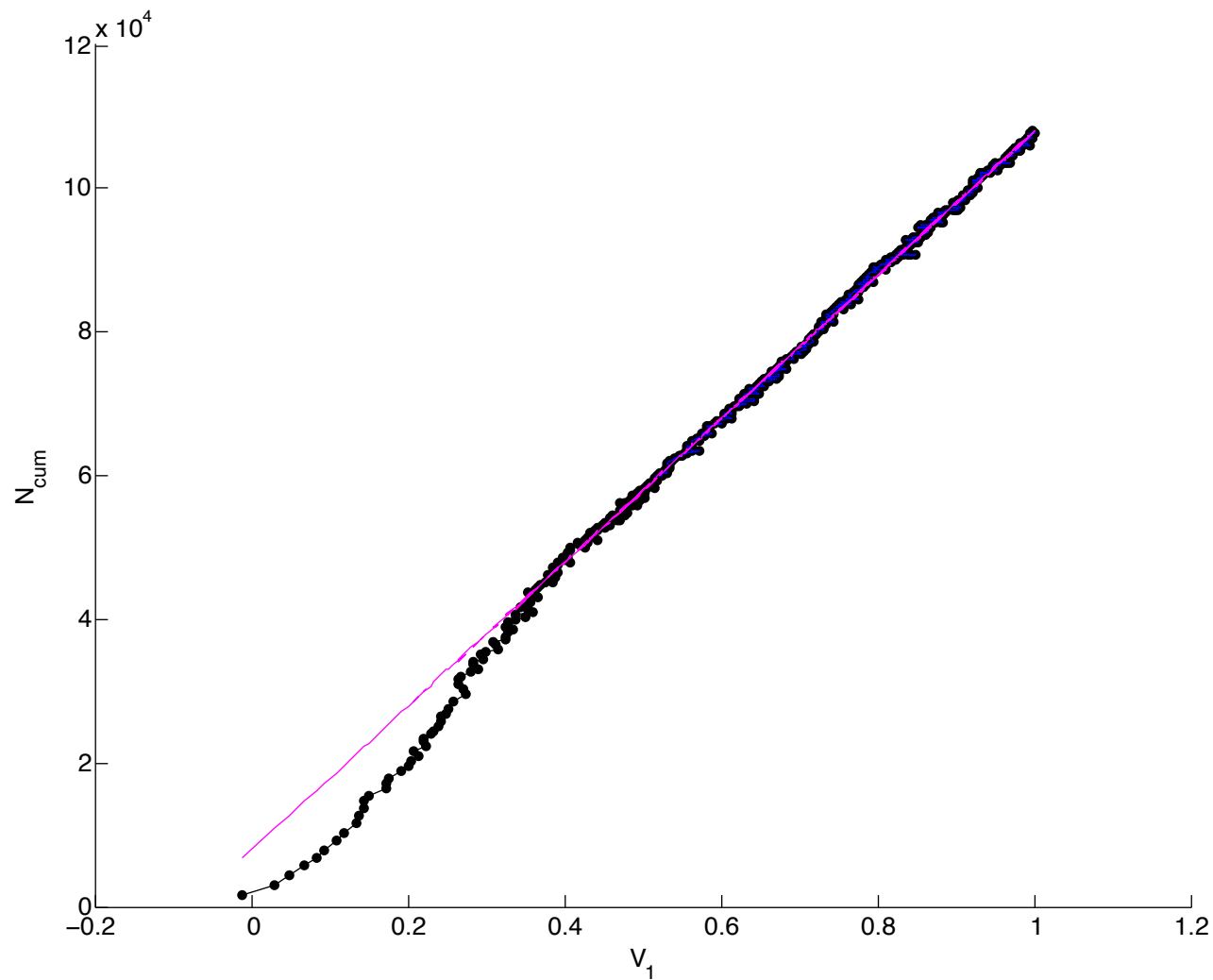
$R_L$ ,  $t_a$ , and  $R^+$  are 3 constants to be adjusted

# Afterslip Model



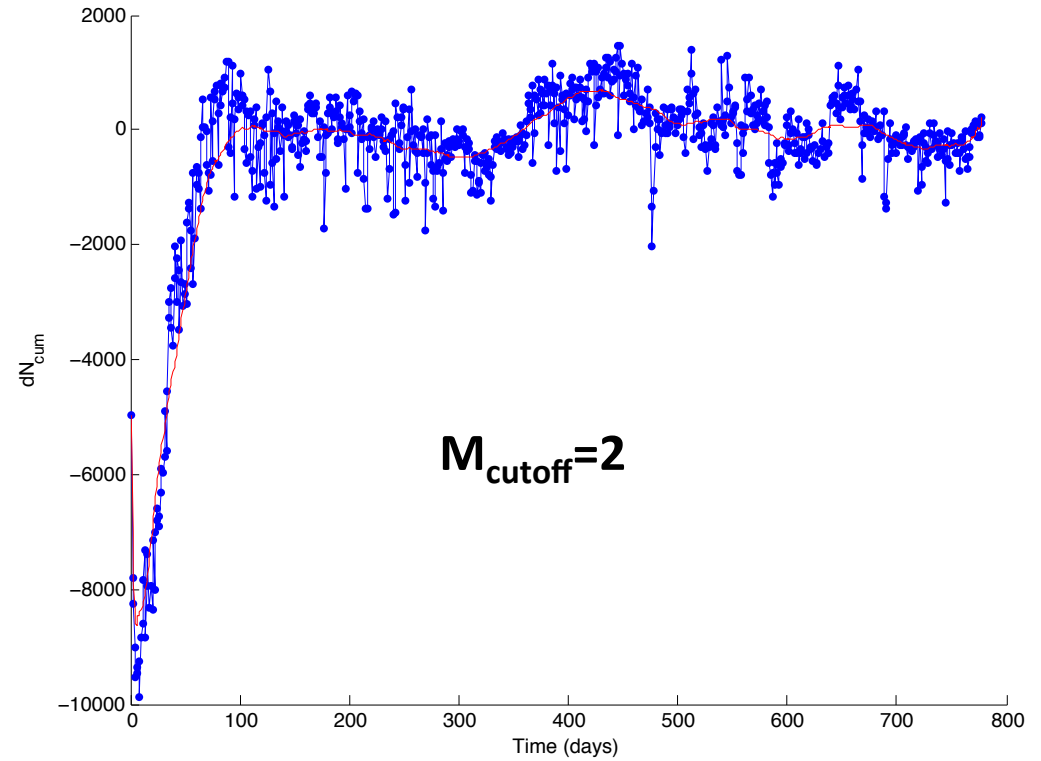
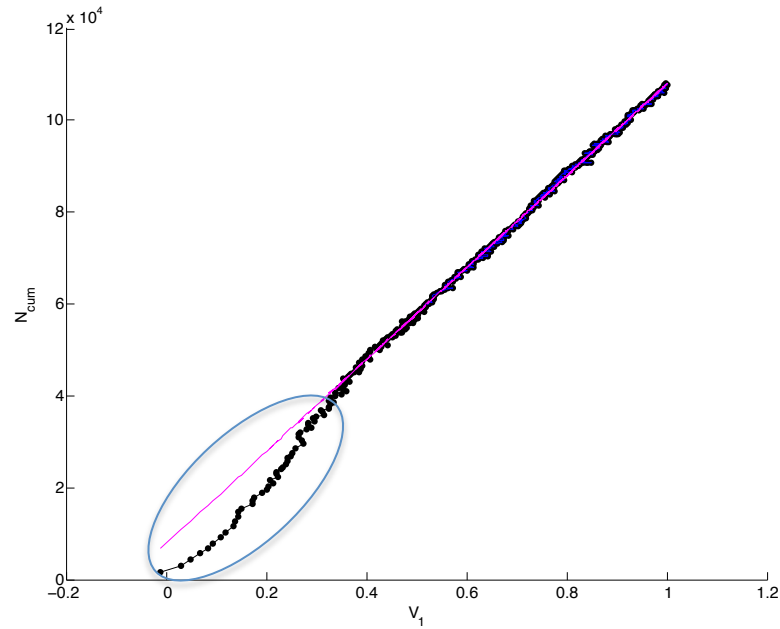
Very good fit to the data for:  
 $R_L=45.71$  evts/day,  $R^+=2160$  evts/day and  $t_a=432.4$  days

# Postseismic Deformation vs. Aftershocks



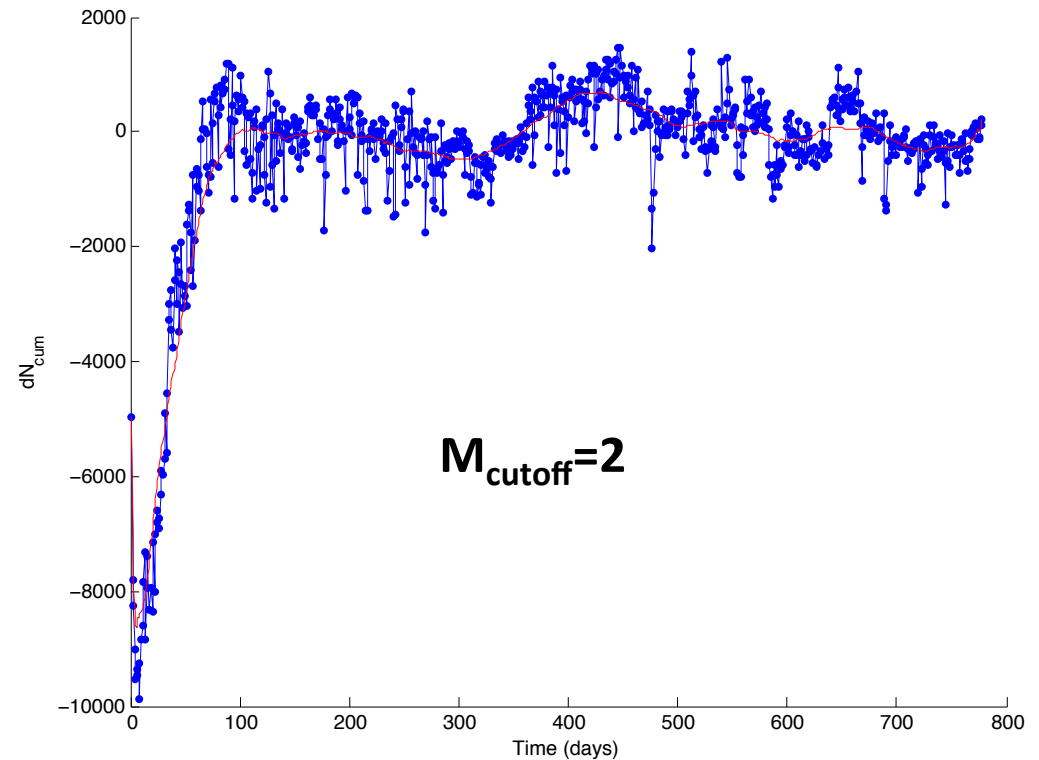
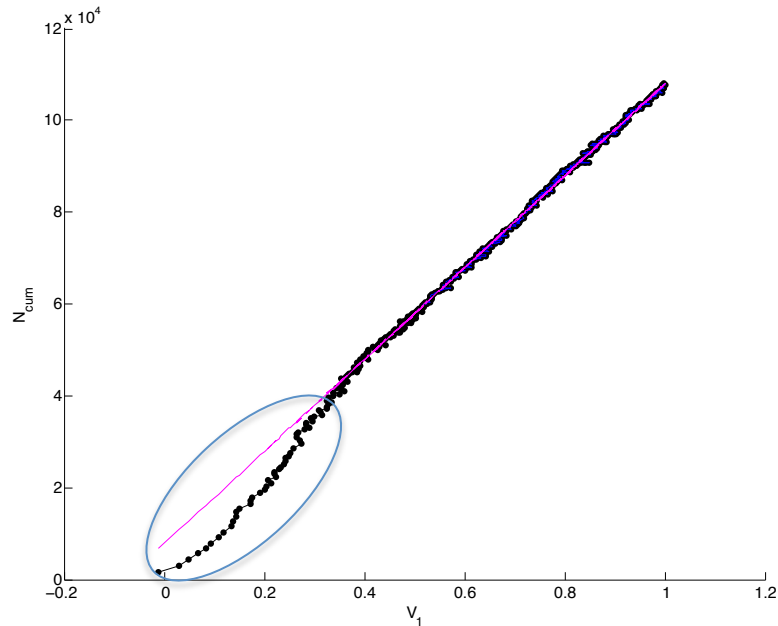
- $N_{\text{cum}} \sim c V_1$
- Deviation at short times that decreases with time

# Postseismic Deformation vs. Aftershocks



Deviation  $dN_{cum}(t) = N_{cum}(t) - cV_1(t)$  from linearity due to???

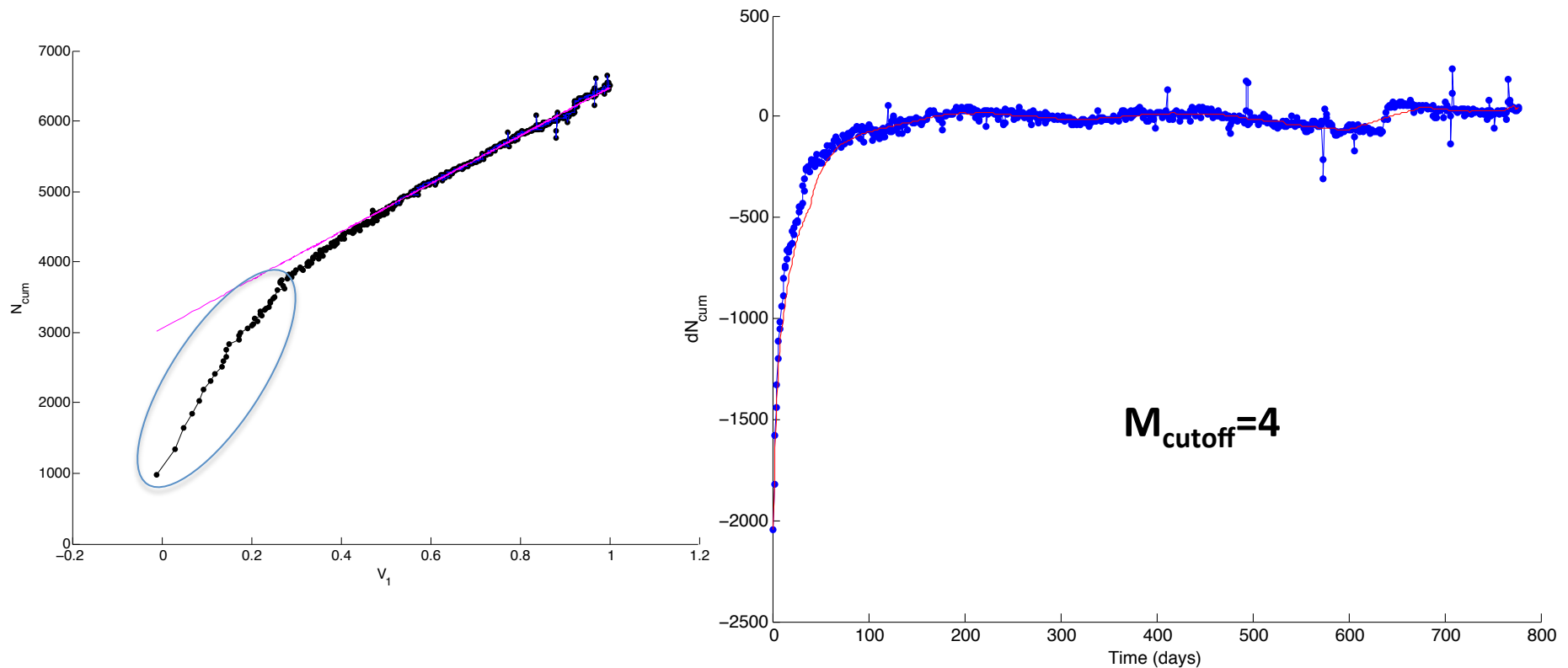
# Postseismic Deformation vs. Aftershocks



Deviation  $dN_{cum}(t) = N_{cum}(t) - cV_1(t)$  from linearity due to:

- Early missing aftershocks?
- The constant  $c$  relating  $N_{cum}$  and  $V_1$  is a function of time?
- Another mechanism controls early aftershocks?

# Postseismic Deformation vs. Aftershocks



- Deviation  $dN_{cum}(t) = N_{cum}(t) - cV_1(t)$  from linearity due to:
- 2000 events missing with  $M_w > 4$  implies that 2 events of  $M_w > 7$  must be missing  $\rightarrow$  The hypothesis of missing aftershocks looks reasonable (e.g., Kiser and Ishii, 2013)



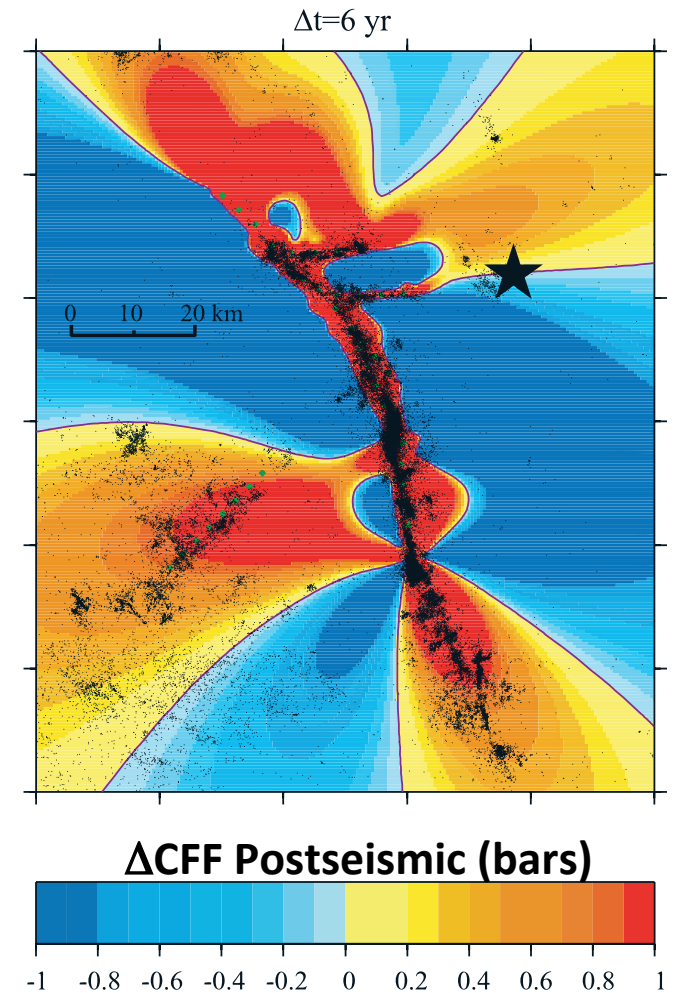
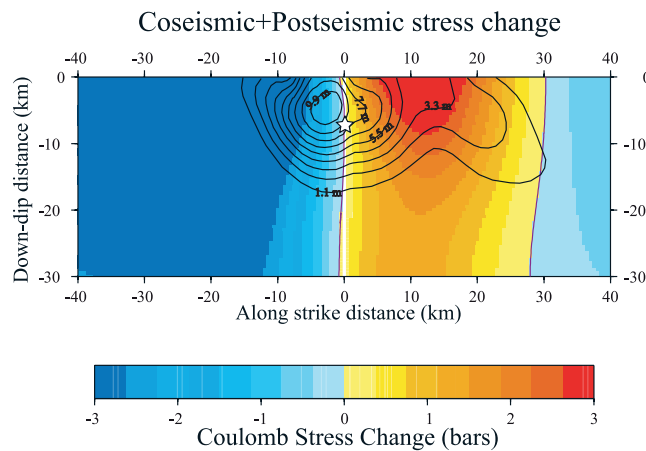
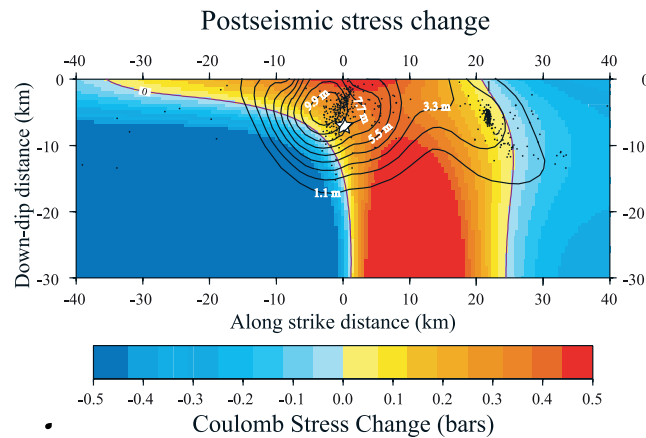
## Conclusions (I)

- In the case of the **Landers** earthquake, the distribution of aftershocks is **consistent** with **postseismic Coulomb stress changes**, assuming that aftershocks occur on faults oriented as the main fault
- In the case of the Landers earthquake, the distribution of aftershocks is **not consistent** with **coseismic Coulomb stress changes**, unless the concept of optimally oriented planes is used

## Conclusions (II)

- Dieterich's model is **equivalent mathematically** to the afterslip seismicity model of *Perfettini and Avouac, 2004*
- The **afterslip model** has the **additional constrain** that **deformation** and **aftershocks** should be **linearly related**, with the idea that afterslip drives aftershocks
- **Seismicity** and **Deformation** seems to follow the **same temporal evolution** in the postseismic phase, in agreement with the hypothesis of the afterslip seismicity model

# Triggering of the Hector Mine Earthquake



The location of the Hector Mine hypocenter has been increased by the postseismic Coulomb stress